

3) Tables

AP<sup>®</sup> CALCULUS AB  
2011 SCORING GUIDELINES

Question 2

$t$ (minutes)	0	2	5	9	10
$H(t)$ (degrees Celsius)	66	60	52	44	43

As a pot of tea cools, the temperature of the tea is modeled by a differentiable function  $H$  for  $0 \leq t \leq 10$ , where time  $t$  is measured in minutes and temperature  $H(t)$  is measured in degrees Celsius. Values of  $H(t)$  at selected values of time  $t$  are shown in the table above.

- (a) Use the data in the table to approximate the rate at which the temperature of the tea is changing at time  $t = 3.5$ . Show the computations that lead to your answer.
- (b) Using correct units, explain the meaning of  $\frac{1}{10} \int_0^{10} H(t) dt$  in the context of this problem. Use a trapezoidal sum with the four subintervals indicated by the table to estimate  $\frac{1}{10} \int_0^{10} H(t) dt$ .
- (c) Evaluate  $\int_0^{10} H'(t) dt$ . Using correct units, explain the meaning of the expression in the context of this problem.
- (d) At time  $t = 0$ , biscuits with temperature  $100^\circ\text{C}$  were removed from an oven. The temperature of the biscuits at time  $t$  is modeled by a differentiable function  $B$  for which it is known that  $B'(t) = -13.84e^{-0.173t}$ . Using the given models, at time  $t = 10$ , how much cooler are the biscuits than the tea?

(a) 
$$H'(3.5) \approx \frac{H(5) - H(2)}{5 - 2}$$
$$= \frac{52 - 60}{3} = -2.666 \text{ or } -2.667 \text{ degrees Celsius per minute}$$

1 : answer

- (b)  $\frac{1}{10} \int_0^{10} H(t) dt$  is the average temperature of the tea, in degrees Celsius, over the 10 minutes.

$$\frac{1}{10} \int_0^{10} H(t) dt \approx \frac{1}{10} \left( 2 \cdot \frac{66 + 60}{2} + 3 \cdot \frac{60 + 52}{2} + 4 \cdot \frac{52 + 44}{2} + 1 \cdot \frac{44 + 43}{2} \right)$$
$$= 52.95$$

3 :  $\begin{cases} 1 : \text{meaning of expression} \\ 1 : \text{trapezoidal sum} \\ 1 : \text{estimate} \end{cases}$

(c) 
$$\int_0^{10} H'(t) dt = H(10) - H(0) = 43 - 66 = -23$$

The temperature of the tea drops 23 degrees Celsius from time  $t = 0$  to time  $t = 10$  minutes.

2 :  $\begin{cases} 1 : \text{value of integral} \\ 1 : \text{meaning of expression} \end{cases}$

(d) 
$$B(10) = 100 + \int_0^{10} B'(t) dt = 34.18275; \quad H(10) - B(10) = 8.817$$

The biscuits are 8.817 degrees Celsius cooler than the tea.

3 :  $\begin{cases} 1 : \text{integrand} \\ 1 : \text{uses } B(0) = 100 \\ 1 : \text{answer} \end{cases}$

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**2010 SCORING GUIDELINES**

**Question 2**

$t$ (hours)	0	2	5	7	8
$E(t)$ (hundreds of entries)	0	4	13	21	23

A zoo sponsored a one-day contest to name a new baby elephant. Zoo visitors deposited entries in a special box between noon ( $t = 0$ ) and 8 P.M. ( $t = 8$ ). The number of entries in the box  $t$  hours after noon is modeled by a differentiable function  $E$  for  $0 \leq t \leq 8$ . Values of  $E(t)$ , in hundreds of entries, at various times  $t$  are shown in the table above.

(a) Use the data in the table to approximate the rate, in hundreds of entries per hour, at which entries were being deposited at time  $t = 6$ . Show the computations that lead to your answer.

(b) Use a trapezoidal sum with the four subintervals given by the table to approximate the value of  $\frac{1}{8} \int_0^8 E(t) dt$ .

Using correct units, explain the meaning of  $\frac{1}{8} \int_0^8 E(t) dt$  in terms of the number of entries.

(c) At 8 P.M., volunteers began to process the entries. They processed the entries at a rate modeled by the function  $P$ , where  $P(t) = t^3 - 30t^2 + 298t - 976$  hundreds of entries per hour for  $8 \leq t \leq 12$ . According to the model, how many entries had not yet been processed by midnight ( $t = 12$ )?

(d) According to the model from part (c), at what time were the entries being processed most quickly? Justify your answer.

(a)  $E'(6) \approx \frac{E(7) - E(5)}{7 - 5} = 4$  hundred entries per hour

(b)  $\frac{1}{8} \int_0^8 E(t) dt \approx$   
 $\frac{1}{8} \left( 2 \cdot \frac{E(0) + E(2)}{2} + 3 \cdot \frac{E(2) + E(5)}{2} + 2 \cdot \frac{E(5) + E(7)}{2} + 1 \cdot \frac{E(7) + E(8)}{2} \right)$   
 $= 10.687$  or  $10.688$

$\frac{1}{8} \int_0^8 E(t) dt$  is the average number of hundreds of entries in the box between noon and 8 P.M.

(c)  $23 - \int_8^{12} P(t) dt = 23 - 16 = 7$  hundred entries

(d)  $P'(t) = 0$  when  $t = 9.183503$  and  $t = 10.816497$ .

$t$	$P(t)$
8	0
9.183503	5.088662
10.816497	2.911338
12	8

Entries are being processed most quickly at time  $t = 12$ .

1 : answer

3 :  $\left\{ \begin{array}{l} 1 : \text{trapezoidal sum} \\ 1 : \text{approximation} \\ 1 : \text{meaning} \end{array} \right.$

2 :  $\left\{ \begin{array}{l} 1 : \text{integral} \\ 1 : \text{answer} \end{array} \right.$

3 :  $\left\{ \begin{array}{l} 1 : \text{considers } P'(t) = 0 \\ 1 : \text{identifies candidates} \\ 1 : \text{answer with justification} \end{array} \right.$

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2015 SCORING GUIDELINES**

**Question 3**

$t$ (minutes)	0	12	20	24	40
$v(t)$ (meters per minute)	0	200	240	-220	150

Johanna jogs along a straight path. For  $0 \leq t \leq 40$ , Johanna's velocity is given by a differentiable function  $v$ . Selected values of  $v(t)$ , where  $t$  is measured in minutes and  $v(t)$  is measured in meters per minute, are given in the table above.

- (a) Use the data in the table to estimate the value of  $v'(16)$ .
- (b) Using correct units, explain the meaning of the definite integral  $\int_0^{40} |v(t)| dt$  in the context of the problem.

Approximate the value of  $\int_0^{40} |v(t)| dt$  using a right Riemann sum with the four subintervals indicated in the table.

- (c) Bob is riding his bicycle along the same path. For  $0 \leq t \leq 10$ , Bob's velocity is modeled by  $B(t) = t^3 - 6t^2 + 300$ , where  $t$  is measured in minutes and  $B(t)$  is measured in meters per minute.

Find Bob's acceleration at time  $t = 5$ .

- (d) Based on the model  $B$  from part (c), find Bob's average velocity during the interval  $0 \leq t \leq 10$ .

(a)  $v'(16) \approx \frac{240 - 200}{20 - 12} = 5 \text{ meters/min}^2$

1 : approximation

- (b)  $\int_0^{40} |v(t)| dt$  is the total distance Johanna jogs, in meters, over the time interval  $0 \leq t \leq 40$  minutes.

3 :  $\begin{cases} 1 : \text{explanation} \\ 1 : \text{right Riemann sum} \\ 1 : \text{approximation} \end{cases}$

$$\begin{aligned} \int_0^{40} |v(t)| dt &\approx 12 \cdot |v(12)| + 8 \cdot |v(20)| + 4 \cdot |v(24)| + 16 \cdot |v(40)| \\ &= 12 \cdot 200 + 8 \cdot 240 + 4 \cdot 220 + 16 \cdot 150 \\ &= 2400 + 1920 + 880 + 2400 \\ &= 7600 \text{ meters} \end{aligned}$$

- (c) Bob's acceleration is  $B'(t) = 3t^2 - 12t$ .  
 $B'(5) = 3(25) - 12(5) = 15 \text{ meters/min}^2$

2 :  $\begin{cases} 1 : \text{uses } B'(t) \\ 1 : \text{answer} \end{cases}$

(d) Avg vel =  $\frac{1}{10} \int_0^{10} (t^3 - 6t^2 + 300) dt$

$$\begin{aligned} &= \frac{1}{10} \left[ \frac{t^4}{4} - 2t^3 + 300t \right]_0^{10} \\ &= \frac{1}{10} \left[ \frac{10000}{4} - 2000 + 3000 \right] = 350 \text{ meters/min} \end{aligned}$$

3 :  $\begin{cases} 1 : \text{integral} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

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**2012 SCORING GUIDELINES**

**Question 1**

$t$ (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

The temperature of water in a tub at time  $t$  is modeled by a strictly increasing, twice-differentiable function  $W$ , where  $W(t)$  is measured in degrees Fahrenheit and  $t$  is measured in minutes. At time  $t = 0$ , the temperature of the water is  $55^\circ\text{F}$ . The water is heated for 30 minutes, beginning at time  $t = 0$ . Values of  $W(t)$  at selected times  $t$  for the first 20 minutes are given in the table above.

- (a) Use the data in the table to estimate  $W'(12)$ . Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.
- (b) Use the data in the table to evaluate  $\int_0^{20} W'(t) dt$ . Using correct units, interpret the meaning of  $\int_0^{20} W'(t) dt$  in the context of this problem.
- (c) For  $0 \leq t \leq 20$ , the average temperature of the water in the tub is  $\frac{1}{20} \int_0^{20} W(t) dt$ . Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate  $\frac{1}{20} \int_0^{20} W(t) dt$ . Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.
- (d) For  $20 \leq t \leq 25$ , the function  $W$  that models the water temperature has first derivative given by  $W'(t) = 0.4\sqrt{t} \cos(0.06t)$ . Based on the model, what is the temperature of the water at time  $t = 25$ ?

(a) 
$$W'(12) \approx \frac{W(15) - W(9)}{15 - 9} = \frac{67.9 - 61.8}{6}$$

$$= 1.017 \text{ (or } 1.016)$$

The water temperature is increasing at a rate of approximately  $1.017^\circ\text{F}$  per minute at time  $t = 12$  minutes.

(b) 
$$\int_0^{20} W'(t) dt = W(20) - W(0) = 71.0 - 55.0 = 16$$

The water has warmed by  $16^\circ\text{F}$  over the interval from  $t = 0$  to  $t = 20$  minutes.

(c) 
$$\frac{1}{20} \int_0^{20} W(t) dt \approx \frac{1}{20} (4 \cdot W(0) + 5 \cdot W(4) + 6 \cdot W(9) + 5 \cdot W(15))$$

$$= \frac{1}{20} (4 \cdot 55.0 + 5 \cdot 57.1 + 6 \cdot 61.8 + 5 \cdot 67.9)$$

$$= \frac{1}{20} \cdot 1215.8 = 60.79$$

This approximation is an underestimate, because a left Riemann sum is used and the function  $W$  is strictly increasing.

(d) 
$$W(25) = 71.0 + \int_{20}^{25} W'(t) dt$$

$$= 71.0 + 2.043155 = 73.043$$

2 :  $\left\{ \begin{array}{l} 1 : \text{estimate} \\ 1 : \text{interpretation with units} \end{array} \right.$

2 :  $\left\{ \begin{array}{l} 1 : \text{value} \\ 1 : \text{interpretation with units} \end{array} \right.$

3 :  $\left\{ \begin{array}{l} 1 : \text{left Riemann sum} \\ 1 : \text{approximation} \\ 1 : \text{underestimate with reason} \end{array} \right.$

2 :  $\left\{ \begin{array}{l} 1 : \text{integral} \\ 1 : \text{answer} \end{array} \right.$

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**Question 5**

$x$	2	3	5	8	13
$f(x)$	1	4	-2	3	6

Let  $f$  be a function that is twice differentiable for all real numbers. The table above gives values of  $f$  for selected points in the closed interval  $2 \leq x \leq 13$ .

- (a) Estimate  $f'(4)$ . Show the work that leads to your answer.
- (b) Evaluate  $\int_2^{13} (3 - 5f'(x)) dx$ . Show the work that leads to your answer.
- (c) Use a left Riemann sum with subintervals indicated by the data in the table to approximate  $\int_2^{13} f(x) dx$ . Show the work that leads to your answer.
- (d) Suppose  $f'(5) = 3$  and  $f''(x) < 0$  for all  $x$  in the closed interval  $5 \leq x \leq 8$ . Use the line tangent to the graph of  $f$  at  $x = 5$  to show that  $f(7) \leq 4$ . Use the secant line for the graph of  $f$  on  $5 \leq x \leq 8$  to show that  $f(7) \geq \frac{4}{3}$ .

(a)  $f'(4) \approx \frac{f(5) - f(3)}{5 - 3} = -3$

(b) 
$$\int_2^{13} (3 - 5f'(x)) dx = \int_2^{13} 3 dx - 5 \int_2^{13} f'(x) dx$$

$$= 3(13 - 2) - 5(f(13) - f(2)) = 8$$

(c) 
$$\int_2^{13} f(x) dx \approx f(2)(3 - 2) + f(3)(5 - 3)$$

$$+ f(5)(8 - 5) + f(8)(13 - 8) = 18$$

- (d) An equation for the tangent line is  $y = -2 + 3(x - 5)$ .  
Since  $f''(x) < 0$  for all  $x$  in the interval  $5 \leq x \leq 8$ , the line tangent to the graph of  $y = f(x)$  at  $x = 5$  lies above the graph for all  $x$  in the interval  $5 < x \leq 8$ .

Therefore,  $f(7) \leq -2 + 3 \cdot 2 = 4$ .

An equation for the secant line is  $y = -2 + \frac{5}{3}(x - 5)$ .

Since  $f''(x) < 0$  for all  $x$  in the interval  $5 \leq x \leq 8$ , the secant line connecting  $(5, f(5))$  and  $(8, f(8))$  lies below the graph of  $y = f(x)$  for all  $x$  in the interval  $5 < x < 8$ .

Therefore,  $f(7) \geq -2 + \frac{5}{3} \cdot 2 = \frac{4}{3}$ .

1 : answer

2 :  $\left\{ \begin{array}{l} 1 : \text{uses Fundamental Theorem} \\ \quad \text{of Calculus} \\ 1 : \text{answer} \end{array} \right.$

2 :  $\left\{ \begin{array}{l} 1 : \text{left Riemann sum} \\ 1 : \text{answer} \end{array} \right.$

4 :  $\left\{ \begin{array}{l} 1 : \text{tangent line} \\ 1 : \text{shows } f(7) \leq 4 \\ 1 : \text{secant line} \\ 1 : \text{shows } f(7) \geq \frac{4}{3} \end{array} \right.$

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**Question 3**

$t$ (minutes)	0	1	2	3	4	5	6
$C(t)$ (ounces)	0	5.3	8.8	11.2	12.8	13.8	14.5

Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time  $t$ ,  $0 \leq t \leq 6$ , is given by a differentiable function  $C$ , where  $t$  is measured in minutes. Selected values of  $C(t)$ , measured in ounces, are given in the table above.

- (a) Use the data in the table to approximate  $C'(3.5)$ . Show the computations that lead to your answer, and indicate units of measure.
- (b) Is there a time  $t$ ,  $2 \leq t \leq 4$ , at which  $C'(t) = 2$ ? Justify your answer.
- (c) Use a midpoint sum with three subintervals of equal length indicated by the data in the table to approximate the value of  $\frac{1}{6} \int_0^6 C(t) dt$ . Using correct units, explain the meaning of  $\frac{1}{6} \int_0^6 C(t) dt$  in the context of the problem.
- (d) The amount of coffee in the cup, in ounces, is modeled by  $B(t) = 16 - 16e^{-0.4t}$ . Using this model, find the rate at which the amount of coffee in the cup is changing when  $t = 5$ .

(a)  $C'(3.5) \approx \frac{C(4) - C(3)}{4 - 3} = \frac{12.8 - 11.2}{1} = 1.6$  ounces/min

2 :  $\begin{cases} 1 : \text{approximation} \\ 1 : \text{units} \end{cases}$

(b)  $C$  is differentiable  $\Rightarrow C$  is continuous (on the closed interval)  
 $\frac{C(4) - C(2)}{4 - 2} = \frac{12.8 - 8.8}{2} = 2$

Therefore, by the Mean Value Theorem, there is at least one time  $t$ ,  $2 < t < 4$ , for which  $C'(t) = 2$ .

2 :  $\begin{cases} 1 : \frac{C(4) - C(2)}{4 - 2} \\ 1 : \text{conclusion, using MVT} \end{cases}$

(c)  $\frac{1}{6} \int_0^6 C(t) dt \approx \frac{1}{6} [2 \cdot C(1) + 2 \cdot C(3) + 2 \cdot C(5)]$   
 $= \frac{1}{6} (2 \cdot 5.3 + 2 \cdot 11.2 + 2 \cdot 13.8)$   
 $= \frac{1}{6} (60.6) = 10.1$  ounces

3 :  $\begin{cases} 1 : \text{midpoint sum} \\ 1 : \text{approximation} \\ 1 : \text{interpretation} \end{cases}$

$\frac{1}{6} \int_0^6 C(t) dt$  is the average amount of coffee in the cup, in ounces, over the time interval  $0 \leq t \leq 6$  minutes.

(d)  $B'(t) = -16(-0.4)e^{-0.4t} = 6.4e^{-0.4t}$   
 $B'(5) = 6.4e^{-0.4(5)} = \frac{6.4}{e^2}$  ounces/min

2 :  $\begin{cases} 1 : B'(t) \\ 1 : B'(5) \end{cases}$