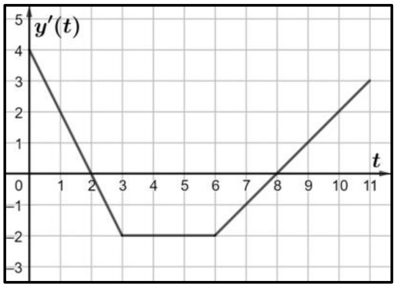
Calculus BC - 2021 AP Live Review Session 3 — SOLUTIONS
Everything You Need to Know About Parametrics



t	0	2	5	7	11
$x'(t)$	1	6	0	3	-2

1: At time t, the position of a particle moving in the xy − plane is given by the parametric functions $(x(t), y(t))$. Selected values of $x'(t)$ are shown in the table above. The graph of $y'(t)$, shown above, consists of three line segments. At time $t = 0$, the particle is at position $(4, -2)$.

a) For $0 \le t \le 11$, it is known that the particle is at rest exactly once. At what time t is the particle at rest? Give a reason for your answer.

$y'(t) = 0$ when $t = 2$ and $t = 8$.
$x'(t) = 0$ when $t = 5$ and, according to the IVT, some time between $t = 7$ and $t = 11$.
Because the particle is at rest *exactly* once, that time must be 8.

b) Find the slope of the line tangent to the path of the particle at time $t = 7$.

The slope of the tangent is $\dfrac{dy/dt}{dx/dt}$.

$$\left.\frac{dy/dt}{dx/dt}\right|_{t=7} = \frac{-1}{3}$$

c) Using a left Riemann sum with the four subintervals indicated in the table above, approximate the x coordinate of the particle at time $t = 11$.

$$x(11) = x(0) + \int_0^{11} x'(t)\,dt$$
$$\approx 4 + 2\cdot(1) + 3\cdot(6) + 2\cdot(0) + 4\cdot(3)$$
$$\approx 4 + 2 + 18 + 12 = 36$$

d) Find the speed of the particle at time $t = 7$. Describe the direction of motion of the particle at time $t = 7$.

The speed of the particle at time t is $\sqrt{(x'(t))^2 + (y'(t))^2}$.

$$\text{speed}\big|_{t=7} = \sqrt{(x'(7))^2 + (y'(7))^2} = \sqrt{3^2 + (-1)^2} = \sqrt{10}$$

$x'(7) = 3,\ y'(7) = -1$
The particle is moving to the right at $t = 7$ because $x'(7) > 0$
and moving downward at $t = 7$ because $y'(7) < 0$.

e) Find the average velocity of the particle in the vertical diretion on the interval $0 \le t \le 11$.

Using Geometric Shapes

$$\text{average}_{\text{vert vel}} = \frac{1}{11-0}\int_0^{11} y'(t)\,dt$$
$$= \frac{1}{11}\left(\int_0^3 (-2t+4)\,dt + \int_3^6 (-2)\,dt + \int_6^{11} (t-8)\,dt\right)$$
$$= \frac{1}{11}\left(\frac{1}{2}(2)(4) - \frac{1}{2}(6+3)(2) + \frac{1}{2}(3)(3)\right)$$
$$= \frac{1}{11}\left(4 - 9 + \frac{9}{2}\right)$$
$$= \frac{1}{11}\left(-\frac{1}{2}\right)$$
$$= -\frac{1}{22}$$

OR

Using Definite Integrals

$$\text{average}_{\text{vert vel}} = \frac{1}{11-0}\int_0^{11} y'(t)\,dt$$
$$= \frac{1}{11}\left(\int_0^3 (-2t+4)\,dt + \int_3^6 (-2)\,dt + \int_6^{11} (t-8)\,dt\right)$$
$$= \frac{1}{11}\left(\left(-t^2+4t\right)\Big|_0^3 + \left(-2t\right)\Big|_3^6 + \left(\frac{t^2}{2} - 8t\right)\Big|_6^{11}\right)$$
$$= \frac{1}{11}(-9+12) + (-12+6) + \frac{121}{2} - 88 - (18-48)$$
$$= \frac{1}{11}\left(-3 + \frac{121}{2} - 58\right) = \frac{1}{11}\left(\frac{121}{2} - \frac{122}{2}\right) = -\frac{1}{22}$$

2: For $t \geq 0$, a particle is moving along a curve so that its position at time t is $(x(t), y(t))$. It is known that $\frac{dx}{dt} = t - 21\sin(2^{0.34t})$ and $y(t) = 13 - 1.8\ln(2.7t^2 - 1.5t + 2) + 2.4t\cos(0.5t)$. At time $= 3$, the particle is at position $(-3, 1)$.

a) Find the speed of the particle at time $t = 2$.

$$\text{speed}\Big|_{t=2} = \sqrt{(x'(2))^2 + (y'(2))^2} \approx 19.144 \text{ or } 19.145$$

b) Find the position of the particle at time $t = 8$.

$$x(8) = x(3) + \int_3^8 x'(t)\,dt \qquad y(8) \approx -8.7165$$

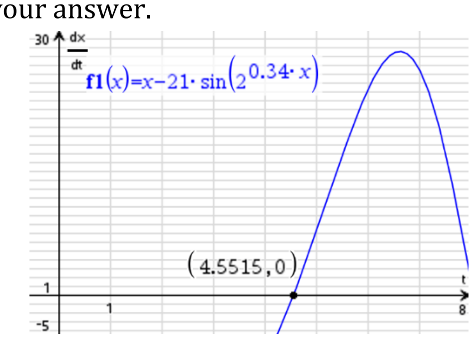
$$\approx -3 + 44.666$$

$$\approx 41.666 \qquad \text{position } (41.666, -8.717)$$

c) Find the distance the particle travels from time $t = 0$ to $t = 1$.

$$\text{distance}\Big|_{0 \le t \le 1} = \int_0^1 \sqrt{(x'(t))^2 + (y'(t))^2}\,dt = 18.512$$

d) For $0 \leq t \leq 8$, at what time t is the particle farthest to the right? Justify your answer.



$\frac{dx}{dt} = 0$ when $t \approx 4.551$, however, we can omit this value from the table because $x(t)$ reaches a relative minimum at that value as $\frac{dx}{dt}$ changes from negative to positive at $t \approx 4.551$.

t	$x(t)$
0	52.357
8	41.666

The particle is farthest to the right at time $t = 0$.

$$-3 - \int_0^3 xd(t)\,dt \quad 52.3673$$

$$-3 + \int_3^8 xd(t)\,dt \quad 41.666$$

1. The position of a particle moving in the xy-plane is given by the parametric equations $(x(t), y(t))$ where $\frac{dx}{dt} = 3t^3 - 6t^2$ and $\frac{dy}{dt} = t^2 - 10t + 16$. At which of the following times is the particle at rest?

(A) $t = \frac{4}{3}$ **(B)** $t = 2$ **(C)** $t = 5$ **(D)** $t = 8$

$x'(t) = 0$ when $3t^3 - 6t^2 = 0 \to 3t^2(t-2) = 0 \to t = 0, t = 2$

$y'(t) = 0$ when $t^2 - 10t + 16 = 0 \to (t-8)(t-2) = 0 \to t = 2, t = 8$

2. For $t \geq 0$, a particle moves in the xy-plane. The velocity vector for the particle is given by $v(t) = \langle e^{3t}, \cos(t^2)\rangle$. Which of the following gives the acceleration vector of the particle at time $t = 4$?

(A) $\langle e^{12}, \cos(16)\rangle$

(B) $\langle 3e^{12,8\cos}(16)\rangle$

(C) $\langle e^{12}, -\sin(16)\rangle$

(D) $\langle 3e^{12}, -8\sin(16)\rangle$

$$\mathbf{a}(t) = \left\langle d\frac{\left(e^{3t}\right)}{dt}, d\frac{\left(\cos\left(t^2\right)\right)}{dt}\right\rangle$$

$$= \left\langle 3e^{3t}, -2t\sin\left(t^2\right)\right\rangle$$

$$\mathbf{a}(4) = \left\langle 3e^{12}, -8\sin(16)\right\rangle$$

3. The position of a particle moving in a plane is given by $x(t) = t^2 - 2t - 15$ and $y(t) = 3t^2 - 12t$. At what time t, is the line tangent to the path of the particle vertical?

(A) $t = 1$ **(B)** $t = 2$ **(C)** $t = 5$ **(D)** $t = 6$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{6t-12}{2t-2}$$

The tangent line is vertical when $2t - 2 = 0 \to t = 1$

4. An object moves in the xy-plane so that its position at time t is given by $(x(t), y(t))$, where $\frac{dx}{dt} = \ln(t^2 - t + 2)$ and $y(t) = -\frac{3}{e^{2t}}$. Which of the following correctly describes the direction of motion of the object when $t = 1$?

(A) up and to the left

(B) up and to the right

(C) down and to the left

(D) down and to the right

$$\left.\frac{dx}{dt}\right|_{t=1} = \ln(1-1+2) = \ln 2 \qquad \left.\frac{dy}{dt}\right|_{t=1} = -\frac{3(-2)}{e^{2t}} = \frac{6}{e^{2t}}$$

The particle moves to the right because $\frac{dx}{dt} > 0$.

The particle moves up because $\frac{dy}{dt} > 0$.

5. For $t \geq 0$, the position of a particle moving along a curve in the xy-plane is defined by the parametric equations $x(t) = 1.53\cos^2(t^3 - 4t)$ and $y(t) = -\dfrac{14t}{t^2+5}$. Which of the following gives the position of particle the first time the speed of the particle is 7?

(**A**) $(1.371, 3.103)$ (**B**) $(0.681, 1.815)$ (**C**) $(0.256, -3.071)$ (**D**) $(3.819, -0.211)$

$$\text{speed} = \sqrt{\left(\frac{d}{dt}\left(1.53\cos^2(t^3-4t)\right)\right)^2 + \left(\frac{d}{dt}\left(-\frac{14}{t^2+5}\right)\right)^2} = 7$$

at $t \approx 1.83691$ the first time speed is 7

$x(1.8369) \approx 0.2558$ $y(1.8369) \approx -3.0709$