Day 4 Homework

Use your calculator on problems 10 and 13c only.

1. If $x = t^2 - 1$ and $y = e^{t^3}$, find $\dfrac{dy}{dx}$.

2. If a particle moves in the *xy*-plane so that at any time $t > 0$, its position vector is $\left\langle \ln\left(t^2 + 5t\right), 3t^2 \right\rangle$, find its velocity vector at time $t = 2$.

3. A particle moves in the *xy*-plane so that at any time *t*, its coordinates are given by $x = t^5 - 1$ and $y = 3t^4 - 2t^3$. Find its acceleration vector at $t = 1$.

4. If a particle moves in the *xy*-plane so that at time *t* its position vector is $\left\langle \sin\left(3t - \dfrac{\pi}{2}\right), 3t^2 \right\rangle$, find the velocity vector at time $t = \dfrac{\pi}{2}$.

5. A particle moves on the curve $y = \ln x$ so that its *x*-component has derivative $x'(t) = t + 1$ for $t \geq 0$. At time $t = 0$, the particle is at the point (1, 0). Find the position of the particle at time $t = 1$.

6. A particle moves in the *xy*-plane in such a way that its velocity vector is $\langle 1 + t, t^3 \rangle$. If the position vector at $t = 0$ is $\langle 5, 0 \rangle$, find the position of the particle at $t = 2$.

7. A particle moves along the curve $xy = 10$. If $x = 2$ and $\dfrac{dy}{dt} = 3$, what is the value of $\dfrac{dx}{dt}$?

8. The position of a particle moving in the *xy*-plane is given by the parametric equations $x = t^3 - \dfrac{3}{2}t^2 - 18t + 5$ and $y = t^3 - 6t^2 + 9t + 4$. For what value(s) of *t* is the particle at rest?

9. A curve *C* is defined by the parametric equations $x = t^3$ and $y = t^2 - 5t + 2$. Write the equation of the line tangent to the graph of *C* at the point (8, –4).

10. A particle moves in the *xy*-plane so that the position of the particle is given by $x(t) = 5t + 3\sin t$ and $y(t) = (8 - t)(1 - \cos t)$. Find the velocity vector at the time when the particle's horizontal position is $x = 25$.

11. The position of a particle at any time $t \geq 0$ is given by $x(t) = t^2 - 3$ and $y(t) = \dfrac{2}{3}t^3$.

(a) Find the magnitude of the velocity vector at time $t = 5$.

(b) Find the total distance traveled by the particle from $t = 0$ to $t = 5$.

(c) Find $\dfrac{dy}{dx}$ as a function of x.

12. Point $P(x, y)$ moves in the xy-plane in such a way that $\dfrac{dx}{dt} = \dfrac{1}{t+1}$ and $\dfrac{dy}{dt} = 2t$ for $t \geq 0$.

(a) Find the coordinates of P in terms of t given that, when $t = 1$, $x = \ln 2$ and $y = 0$.

(b) Write an equation expressing y in terms of x.

(c) Find the average rate of change of y with respect to x as t varies from 0 to 4.

(d) Find the instantaneous rate of change of y with respect to x when $t = 1$.

13. Consider the curve C given by the parametric equations $x = 2 - 3\cos t$ and $y = 3 + 2\sin t$, for $-\dfrac{\pi}{2} \leq t \leq \dfrac{\pi}{2}$.

(a) Find $\dfrac{dy}{dx}$ as a function of t.

(b) Find the equation of the tangent line at the point where $t = \dfrac{\pi}{4}$.

(c) The curve C intersects the y-axis twice. Approximate the length of the curve between the two y-intercepts.

Answers to Day 4 Homework

1. $$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\frac{d}{dt}\left[e^{t^3}\right]}{\frac{d}{dt}\left[t^2 - 1\right]} = \frac{3t^2 e^{t^3}}{2t} = \frac{3te^{t^3}}{2}.$$

2. $$v(t) = \left\langle \frac{dx}{dt}, \frac{dy}{dt} \right\rangle = \left\langle \frac{2t+5}{t^2+5t}, 6t \right\rangle \text{ so } v(2) = \left\langle \frac{9}{14}, 12 \right\rangle.$$

3. $$v(t) = \left\langle \frac{dx}{dt}, \frac{dy}{dt} \right\rangle = \left\langle 5t^4, 12t^3 - 6t^2 \right\rangle.$$

$$a(t) = \left\langle \frac{d^2x}{dt^2}, \frac{d^2y}{dt^2} \right\rangle = \left\langle 20t^3, 36t^2 - 12t \right\rangle, \text{ so a}(1) = \langle 20, 24 \rangle.$$

4. $$v(t) = \left\langle \frac{dx}{dt}, \frac{dy}{dt} \right\rangle = \left\langle 3\cos\left(3t - \frac{\pi}{2}\right), 6t \right\rangle \text{ so } v\left(\frac{\pi}{2}\right) = \langle 3\cos\pi, 3\pi \rangle = \langle -3, 3\pi \rangle.$$