Day 6: Motion Along a Curve — Vectors (continued)

I don't work any examples on Day 6. The students usually need a little more practice on vectors, but no new material is covered in the Day 6 homework.

Day 6 Homework

Use your calculator only on problems 3–7.

1. The position of a particle at any time $t \geq 0$ is given by $x(t) = t^2 - 2,\ y(t) = \frac{2}{3}t^3$.

(a) Find the magnitude of the velocity vector at $t = 2$.

(b) Set up an integral expression to find the total distance traveled by the particle from $t = 0$ to $t = 4$.

(c) Find $\frac{dy}{dx}$ as a function of x.

(d) At what time t is the particle on the y-axis? Find the acceleration vector at this time.

2. An object moving along a curve in the xy-plane has position $(x(t), y(t))$ at time t with the velocity vector $v(t) = \left(\frac{1}{t+1}, 2t\right)$. At time $t = 1$, the object is at (ln 2, 4).

(a) Find the position vector.

(b) Write an equation for the line tangent to the curve when $t = 1$.

(c) Find the magnitude of the velocity vector when $t = 1$.

(d) At what time $t > 0$ does the line tangent to the particle at $(x(t), y(t))$ have a slope of 12?

3. A particle moving along a curve in the xy-plane has position $(x(t), y(t))$, with $x(t) = 2t + 3\sin t$ and $y(t) = t^2 + 2\cos t$, where $0 \leq t \leq 10$. Find the velocity vector at the time when the particle's vertical position is $y = 7$.

4. A particle moving along a curve in the xy-plane has position $(x(t), y(t))$ at time t with $\frac{dx}{dt} = 1 + \sin(t^3)$. The derivative $\frac{dy}{dt}$ is not explicitly given. For any time t, $t \geq 0$, the line tangent to the curve at $(x(t), y(t))$ has a slope of $t + 3$. Find the acceleration vector of the object at time $t = 2$.

5. An object moving along a curve in the xy-plane has position $\left(x(t), y(t)\right)$ at time t with $\frac{dx}{dt} = \cos\left(e^t\right)$ and $\frac{dy}{dt} = \sin\left(e^t\right)$ for $0 \le t \le 2$. At time $t = 1$, the object is at the point (3, 2).

(a) Find the equation of the tangent line to the curve at the point where $t = 1$.

(b) Find the speed of the object at $t = 1$.

(c) Find the total distance traveled by the object over the time interval $0 \le t \le 2$.

(d) Find the position of the object at time $t = 2$.

6. A particle moving along a curve in the xy-plane has position $\left(x(t), y(t)\right)$ at time t with $\frac{dx}{dt} = \sin\left(t^3 - t\right)$ and $\frac{dy}{dt} = \cos\left(t^3 - t\right)$. At time $t = 3$, the particle is at the point (1, 4).

(a) Find the acceleration vector for the particle at $t = 3$.

(b) Find the equation of the tangent line to the curve at the point where $t = 3$.

(c) Find the magnitude of the velocity vector at $t = 3$.

(d) Find the position of the particle at time $t = 2$.

7. An object moving along a curve in the xy-plane has position $\left(x(t), y(t)\right)$ at time t with $\frac{dy}{dt} = 2 + \sin\left(e^t\right)$. The derivative $\frac{dx}{dt}$ is not explicitly given. At $t = 3$, the object is at the point (4, 5).

(a) Find the y-coordinate of the position at time $t = 1$.

(b) At time $t = 3$, the value of $\frac{dy}{dx}$ is -1.8. Find the value of $\frac{dx}{dt}$ when $t = 3$.

(c) Find the speed of the object at time $t = 3$.