

PTF #AB 39 – Particle Motion Summary 8.2

- Position Function: $s(t)$ or $x(t)$
- Velocity Function: $v(t) = s'(t)$
- Acceleration Function: $a(t) = v'(t) = s''(t)$
- Displacement: $\int_a^b v(t) dt$
- Total Distance: $\int_a^b |v(t)| dt$
- Position of the Particle at time $t = b$: $s(b) = s(a) + \int_a^b v(t) dt$

A particle moves along the x -axis with velocity at time $t \geq 0$ given by $v(t) = -1 + e^{1-t}$.
At time $t = 0$, $s = 2$.

1. Find $a(3)$, $v(3)$ and $s(3)$.

$$v(3) = -1 + e^{1-3} = -1 + e^{-2} \approx -0.865$$

$$a(t) = v'(t) = -e^{1-t}$$

$$a(3) = v'(3) = -e^{-2} \approx -0.135$$

$$s(t) = 2 + \int_0^t (-1 + e^{1-x}) dx$$

$$s(3) = 2 + \int_0^3 v(t) dt$$

$$s(3) \approx 1.583$$

2. Is the speed of the particle increasing at time $t = 3$? Give a reason for your answer.

Yes, because

$$v(3) \cdot a(3) > 0$$

3. Find all values of t for which the particle changes direction. Justify your answer.

$$v(t) = 0 \quad -1 + e^{1-t} = 0$$

$$e^{1-t} = 1$$

$$1 - t = \ln 1$$

$$t = 1$$

The particle changes direction at $t = 1$ because $v(t)$ changes signs

4. Find the displacement and total distance of the particle over the time interval $0 \leq t \leq 3$.

$$\text{displacement} = \int_0^3 v(t) dt$$

$$\approx -0.417$$

$$\text{Total distance} = \int_0^3 |v(t)| dt$$

$$\approx 1.854$$