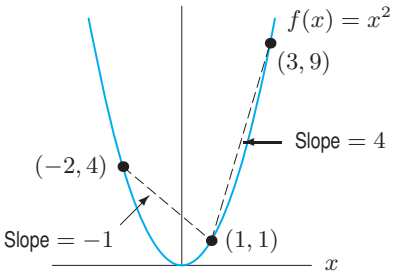

Figure 1.15: Average rate of change of $f(x)$ on an interval is the slope of the dashed line on that interval

Exercises and Problems for Section 1.2

Skill Refresher

In Exercises S1–S10, simplify each expression.

S1. $\dfrac{4-6}{3-2}$

S2. $\dfrac{1-3}{2^2-(-3)^2}$

S3. $\dfrac{-3-(-9)}{-1-2}$

S4. $\dfrac{(1-3^2)-(1-4^2)}{3-4}$

S5. $\dfrac{(\frac{1}{2}-(-4)^2-\left(\frac{1}{2}-(5^2)\right)}{-4-5}$

S6. $2(x+a)-3(x-b)$

S7. $x^2-(2x+a)^2$

S8. $4x^2-(x-b)^2$

S9. $\dfrac{x^2-\frac{3}{4}-\left(y^2-\frac{3}{4}\right)}{x-y}$

S10. $\dfrac{2(x+h)^2-2x^2}{(x+h)-x)}$

Exercises

1. In 2005, you have 40 CDs in your collection. In 2008, you have 120 CDs. In 2012, you have 40. What is the average rate of change in your CD collection's size between

(a) 2005 and 2008? **(b)** 2008 and 2012?

(c) 2005 and 2012?

2. Table 1.10 on page 11 gives the annual sales (in millions) of VCRs and DVD players. What was the average rate of change of annual sales of each of them between

(a) 1998 and 2000? **(b)** 2000 and 2003?

(c) Interpret these results in terms of sales.

3. Table 1.10 on page 11 shows that VCR sales are a function of DVD player sales. Is it an increasing or decreasing function?

4. Table 1.12 shows data for two populations (in hundreds) for five different years. Find the average rate of change of each population over the following intervals.

(a) 1990 to 2000 **(b)** 1995 to 2007

(c) 1990 to 2007

Table 1.12

Year	1990	1992	1995	2000	2007
P_1	53	63	73	83	93
P_2	85	80	75	70	65

Exercises 5–9 use Figure 1.16.

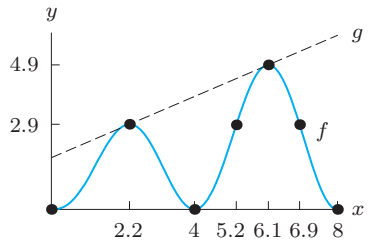

Figure 1.16

5. Find the average rate of change of f for $2.2 \le x \le 6.1$.

6. Give two different intervals on which $\Delta f(x)/\Delta x = 0$.

7. What is the average rate of change of g between $x = 2.2$ and $x = 6.1$?

8. What is the relation between the average rate of change of f and the average rate of change of g between $x = 2.2$ and $x = 6.1$?

9. Is the rate of change of f positive or negative on the following intervals?

(a) $2.2 \le x \le 4$ **(b)** $5 \le x \le 6$

10. If G is an increasing function, what can you say about $G(3) - G(-1)$?

11. If F is a decreasing function, what can you say about $F(-2)$ compared to $F(2)$?

12. Figure 1.17 shows distance traveled as a function of time.

(a) Find ΔD and Δt between:

(i) $t = 2$ and $t = 5$ (ii) $t = 0.5$ and $t = 2.5$
(iii) $t = 1.5$ and $t = 3$

(b) Compute the rate of change, $\Delta D/\Delta t$, over each of the intervals in part (a), and interpret its meaning.

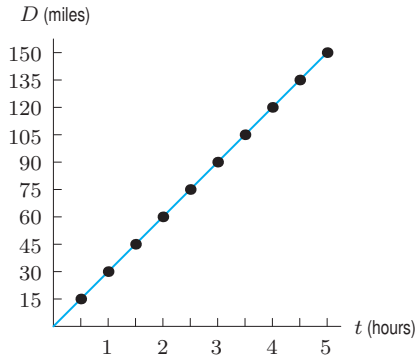

Figure 1.17

Problems

13. Figure 1.18 shows the percent of the side of the moon toward the earth illuminated by the sun at different times during the year 2008. Use the figure to answer the following questions.

(a) Give the coordinates of the points A, B, C, D, E.
(b) Plot the point $F = (15, 60)$ and $G = (60, 15)$. Which point is on the graph?
(c) During which time intervals is the function increasing?
(d) During which time intervals is the function decreasing?

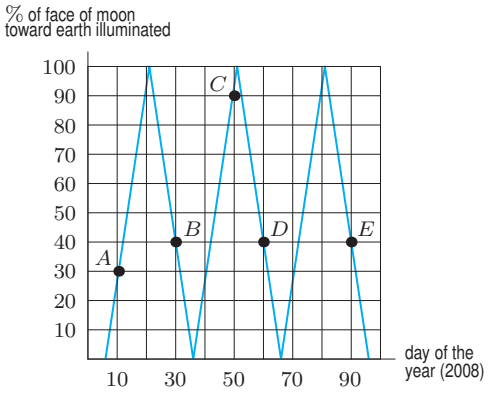

Figure 1.18: Moon phases

14. Imagine you constructed a list of the world record times for a particular event—such as the mile footrace, or the 100-meter freestyle swimming race—in terms of when they were established. Is the world record time a function of the date when it was established? If so, is this function increasing or decreasing? Explain. Could a world record be established twice in the same year? Is the world record time a function of the year it was established?

15. **(a)** What is the average rate of change of $g(x) = 2x - 3$ between the points $(-2, -7)$ and $(3, 3)$?
(b) Based on your answer to part (a), is g increasing or decreasing on the given interval? Explain.
(c) Graph the function and determine over what intervals g is increasing and over what intervals g is decreasing.

16. **(a)** Let $f(x) = 16 - x^2$. Compute each of the following expressions, and interpret each as an average rate of change.

(i) $\dfrac{f(2) - f(0)}{2 - 0}$ (ii) $\dfrac{f(4) - f(2)}{4 - 2}$
(iii) $\dfrac{f(4) - f(0)}{4 - 0}$

(b) Graph $f(x)$. Illustrate each ratio in part (a) by sketching the line segment with the given slope. Over which interval is the average rate of decrease the greatest?

17. Figure 1.19 gives the population of two different towns over a 50-year period of time.

(a) Which town starts (in year $t = 0$) with the most people?
(b) Which town is growing faster over these 50 years?

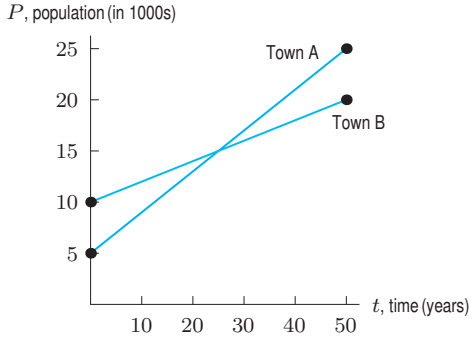

Figure 1.19

18. You have zero dollars now and the average rate of change in your net worth is \$5000 per year. How much money will you have in forty years?

19. The most freakish change in temperature ever recorded was from $-4°$F to $45°$F between 7:30 am and 7:32 am on January 22, 1943 at Spearfish, South Dakota.[8] What was the average rate of change of the temperature for this time period?

20. The surface of the sun has dark areas known as sunspots, that are cooler than the rest of the sun's surface. The number of sunspots fluctuates with time, as shown in Figure 1.20. [9]

(a) Explain how you know the number of sunspots, s, in year t is a function of t.

(b) Approximate the time intervals on which s is an increasing function of t.

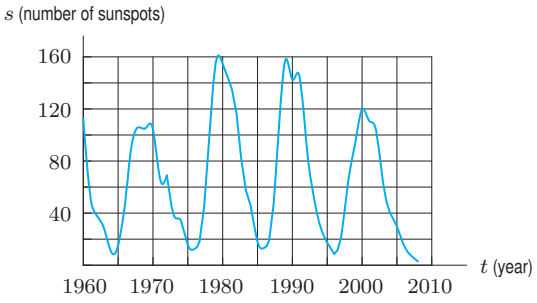

Figure 1.20

21. Table 1.13 shows the number of calories used per minute as a function of body weight for three sports.[10]

(a) Determine the number of calories that a 200-lb person uses in one half-hour of walking.

(b) Who uses more calories, a 120-lb person swimming for one hour or a 220-lb person bicycling for a half-hour?

(c) Does the number of calories used by a person walking increase or decrease as weight increases?

Table 1.13

Activity	100 lb	120 lb	150 lb	170 lb	200 lb	220 lb
Walking	2.7	3.2	4.0	4.6	5.4	5.9
Bicycling	5.4	6.5	8.1	9.2	10.8	11.9
Swimming	5.8	6.9	8.7	9.8	11.6	12.7

22. Because scientists know how much carbon-14 a living organism should have in its tissues, they can measure the amount of carbon-14 present in the tissue of a fossil and then calculate how long it took for the original amount to decay to the current level, thus determining the time of the organism's death. A tree fossil is found to contain 130 μg of carbon-14, and scientists determine from the size of the tree that it would have contained 200 μg of carbon-14 at the time of its death. Using Table 1.11 on page 12, approximately how long ago did the tree die?

23. Find the average rate of change of $f(x) = 3x^2 + 1$ between the points

(a) $(1, 4)$ and $(2, 13)$ **(b)** (j, k) and (m, n)

(c) $(x, f(x))$ and $(x+h, f(x+h))$

24. Figure 1.21 shows the graph of the function $g(x)$.

(a) Estimate $\dfrac{g(4) - g(0)}{4 - 0}$.

(b) The ratio in part (a) is the slope of a line segment joining two points on the graph. Sketch this line segment on the graph.

(c) Estimate $\dfrac{g(b) - g(a)}{b - a}$ for $a = -9$ and $b = -1$.

(d) On the graph, sketch the line segment whose slope is given by the ratio in part (c).

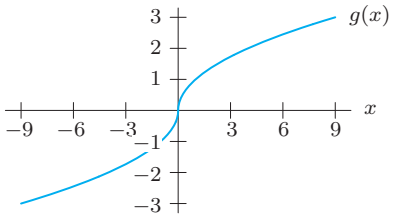

Figure 1.21

25. Table 1.14 gives the amount of garbage, G, in millions of tons, produced[11] in the US in year t.

(a) What is the value of Δt for consecutive entries in this table?

(b) Calculate the value of ΔG for each pair of consecutive entries in this table.

(c) Are all the values of ΔG you found in part (b) the same? What does this tell you?

(d) The function G changed from increasing to decreasing between 2007 and 2008. To what might this be attributed?

Table 1.14

t	1960	1970	1980	1990	2000	2007	2008
G	88.1	121.1	151.6	205.2	239.1	254.6	249.6

[8] *The Guinness Book of Records.* 1995.

[9] ftp://ftp.ngdc.noaa.gov/STP/SOLAR DATA/SUNSPOT NUMBERS/YEARLY.PLT, accessed November 30, 2009.

[10] From *1993 World Almanac.*

[11] http://www.epa.gov/osw/nonhaz/municipal/pubs/msw2008rpt.pdf, accessed November 23, 2009.