

12. Table 1.40 shows men's and women's world records for swimming distances from 50 meters to 1500 meters.<sup>22</sup>
- What values would you add to Table 1.40 to represent the time taken by both men and women to swim 0 meters?
  - Plot men's time against distance, with time  $t$  in seconds on the vertical axis and distance  $d$  in meters on the horizontal axis. It is claimed that a straight line models this behavior well. What is the equation for that line? What does its slope represent? On the same graph, plot women's time against distance and find the equation of the straight line that models this behavior well. Is this line steeper or flatter than the men's line? What does that mean in terms of swimming? What are the values of the vertical intercepts? Do these values have a practical interpretation?
  - On another graph plot the women's times against the men's times, with women's times,  $w$ , on the vertical

axis and men's times,  $m$ , on the horizontal axis. It should look linear. How could you have predicted this linearity from the equations you found in part (b)? What is the slope of this line and how can it be interpreted? A newspaper reporter claims that the women's records are about 8% slower than the men's. Do the facts support this statement? What is the value of the vertical intercept? Does this value have a practical interpretation?

Table 1.40 Men's and women's world swimming records

Distance (m)	50	100	200	400	800	1500
Men (sec)	21.64	47.84	104.06	220.08	458.65	874.56
Women (sec)	24.13	53.62	116.64	243.85	496.22	952.10

## CHAPTER SUMMARY

### • Functions

Definition: a rule which takes certain numbers as inputs and assigns to each input exactly one output number.

Function notation,  $y = f(x)$ .

Use of vertical line test.

### • Average Rate of Change

Average rate of change of  $Q = f(t)$  on  $[a, b]$  is

$$\frac{\Delta Q}{\Delta t} = \frac{f(b) - f(a)}{b - a}.$$

Increasing, decreasing functions; identifying from average rate of change.

### • Linear Functions

Value of  $y$  changes at constant rate.

Tables for linear functions.

### • Formulas for Linear Functions

Slope-intercept form:  $y = b + mx$ .

Point-slope form:  $y - y_0 = m(x - x_0)$ .

Standard form:  $Ax + By + C = 0$ .

### • Properties of Linear Functions

Interpretation of slope, vertical and horizontal intercepts.

Intersection of lines: Solution of equations.

Horizontal and vertical lines.

Parallel lines:  $m_1 = m_2$ .

Perpendicular lines:  $m_1 = -\frac{1}{m_2}$ .

### • Fitting Lines to Data

Linear regression; correlation. Interpolation, extrapolation; dangers of extrapolation.

## REVIEW EXERCISES AND PROBLEMS FOR CHAPTER ONE

### Exercises

In Exercises 1–5 a relationship is given between two quantities. Are both quantities functions of the other one, or is one or neither a function of the other? Explain.

1.  $7w^2 + 5 = z^2$     2.  $y = x^4 - 1$     3.  $m = \sqrt{t}$

4. The number of gallons of gas,  $g$ , at \$2 per gallon and the number of pounds of coffee,  $c$ , at \$10 per pound that can be bought for a total of \$100.

5.

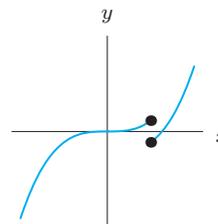


Figure 1.64

<sup>22</sup>Data from *The World Almanac and Book of Facts: 2006*, World Almanac Education Group, Inc., New York, 2006.

6. (a) Which of the graphs in Figure 1.65 represent  $y$  as a function of  $x$ ? (Note that an open circle indicates a point that is not included in the graph; a solid dot indicates a point that is included in the graph.)

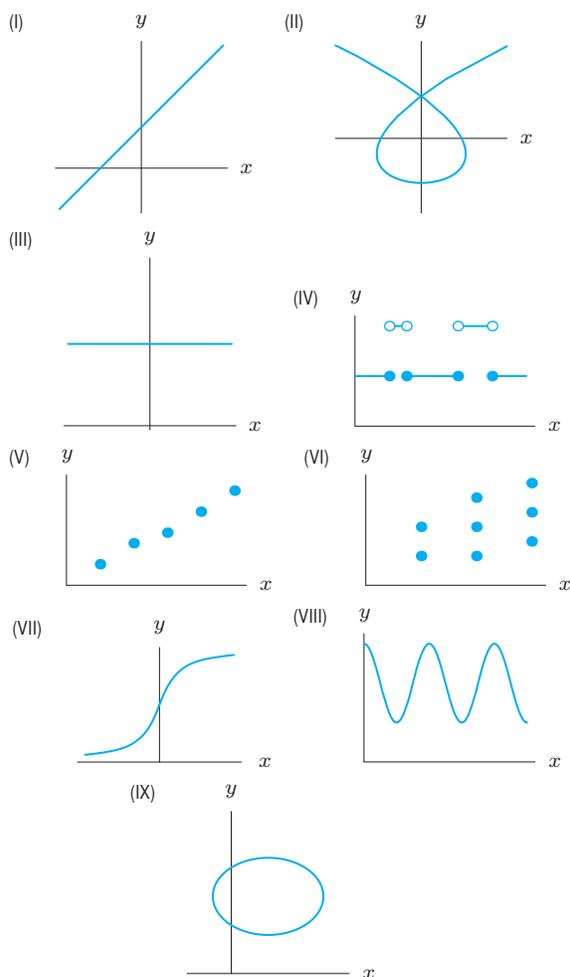


Figure 1.65

- (b) Which of the graphs in Figure 1.65 could represent the following situations? Give reasons.
- SAT Math score versus SAT Verbal score for a small number of students.
  - Total number of daylight hours as a function of the day of the year, shown over a period of several years.
- (c) Among graphs (I)–(IX) in Figure 1.65, find two which could give the cost of train fare as a function of the time of day. Explain the relationship between cost and time for both choices.

7. (a) Make a table of values for  $f(x) = 10/(1 + x^2)$  for  $x = 0, 1, 2, 3$ .  
 (b) What  $x$ -value gives the largest  $f(x)$  value in your table? How could you have predicted this before doing any calculations?

8. Table 1.41 gives the populations of two cities (in thousands) over a 17-year period.

- (a) Find the average rate of change of each population on the following intervals:

- 1990 to 2000
- 1990 to 2007
- 1995 to 2007

- (b) What do you notice about the average rate of change of each population? Explain what the average rate of change tells you about each population.

Table 1.41

Year	1990	1992	1995	2000	2007
$P_1$	42	46	52	62	76
$P_2$	82	80	77	72	65

9. The following tables represent the relationship between the button number,  $N$ , that you push, and the snack,  $S$ , delivered by three different vending machines.<sup>23</sup>

- One of these vending machines is not a good one to use, because  $S$  is not a function of  $N$ . Which one?
- For which vending machine(s) is  $S$  a function of  $N$ ?
- For which of the vending machines is  $N$  not a function of  $S$ ?

Vending Machine #1		Vending Machine #2	
$N$	$S$	$N$	$S$
1	M&Ms	1	M&Ms or dried fruit
2	pretzels	2	pretzels or Hersheys
3	dried fruit		
4	Hersheys	3	Snickers or fat-free cookies
5	fat-free cookies		
6	Snickers		

Vending Machine #3	
$N$	$S$
1	M&Ms
2	M&Ms
3	pretzels
4	dried fruit
5	Hersheys
6	Hersheys
7	fat-free cookies
8	Snickers
9	Snickers

<sup>23</sup>For each  $N$ , vending machine #2 dispenses one or the other product at random.

10. Figure 1.66 shows the average monthly temperature in Albany, New York, over a twelve-month period. (January is month 1.)

- (a) Make a table showing average temperature as a function of the month of the year.  
 (b) What is the warmest month in Albany?  
 (c) Over what interval of months is the temperature increasing? Decreasing?

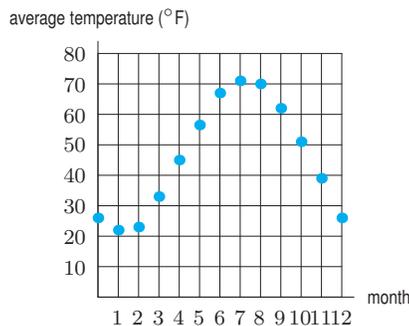


Figure 1.66

11. In 1947, Jesse Owens, the US gold medal track star of the 1930s and 1940s, ran a 100-yard race against a horse. The race, “staged” in Havana, Cuba, is filled with controversy; some say Owens received a head start, others claim the horse was drugged. Owens himself revealed some years later that the starting gun was placed next to the horse’s ear, causing the animal to rear and remain at the gate for a few seconds. Figure 1.67 depicts speeds measured against time for the race.

- (a) How fast were Owens and the horse going at the end of the race?  
 (b) When were the participants both traveling at the same speed?

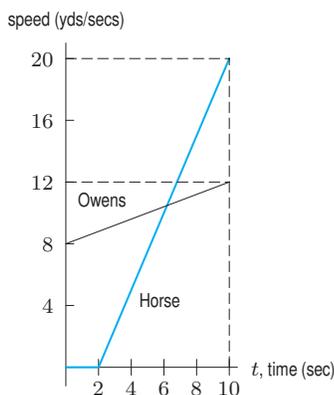


Figure 1.67

In Exercises 12–13, could the table represent a linear function?

12. 

$\lambda$	1	2	3	4	5
$q(\lambda)$	2	4	8	16	32

 13. 

$t$	3	6	9	12	15
$a(t)$	2	4	6	8	10

Problems 14–16 give data from a linear function. Find a formula for the function.

14. 

$x$	200	230	300	320	400
$g(x)$	70	68.5	65	64	60

15. 

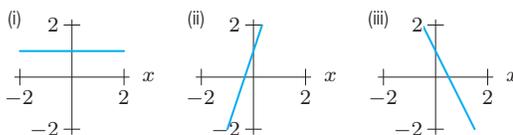
$t$	1.2	1.3	1.4	1.5
$f(t)$	0.736	0.614	0.492	0.37

16. 

$t$	5.2	5.3	5.4	5.5
$f(t)$	73.6	61.4	49.2	37

17. Without a calculator, match the functions (a)–(c) to the graphs (i)–(iii).

- (a)  $f(x) = 3x + 1$       (b)  $g(x) = -2x + 1$   
 (c)  $h(x) = 1$



In Exercises 18–20, which line has the greater

- (a) Slope?      (b)  $y$ -intercept?

18.  $y = -1 + 2x$ ;     $y = -2 + 3x$

19.  $y = 3 + 4x$ ;     $y = 5 - 2x$

20.  $y = \frac{1}{4}x$ ;     $y = 1 - 6x$

Are the lines in Exercises 21–24 perpendicular? Parallel? Neither?

21.  $y = 5x + 2$ ;     $y = 2x + 5$

22.  $y = 14x - 2$ ;     $y = -\frac{1}{14}x + 2$

23.  $y = 3x + 3$ ;     $y = -\frac{1}{3}x + 3$

24.  $7y = 8 + 21x$ ;  $9y = 77 - 3x$

## Problems

In Problems 25–27, use Table 1.42, which gives values of  $v = r(s)$ , the eyewall wind profile of a typical hurricane.<sup>24</sup> The eyewall of a hurricane is the band of clouds that surrounds the eye of the storm. The eyewall wind speed  $v$  (in mph) is a function of the height above the ground  $s$  (in meters).

Table 1.42

$s$	0	100	200	300	400	500
$v$	90	110	116	120	121	122
$s$	600	700	800	900	1000	1100
$v$	121	119	118	117	116	115

25. Evaluate and interpret  $r(300)$ .
26. At what altitudes does the eyewall wind speed appear to equal or exceed 116 mph?
27. At what height is the eyewall wind speed greatest?
28. You are looking at the graph of  $y$ , a function of  $x$ .
  - (a) What is the maximum number of times that the graph can intersect the  $y$ -axis? Explain.
  - (b) Can the graph intersect the  $x$ -axis an infinite number of times? Explain.
29. A bug starts out ten feet from a light, flies closer to the light, then farther away, then closer than before, then farther away. Finally the bug hits the bulb and flies off. Sketch the distance of the bug from the light as a function of time.
30. Although there were 17 women in the Senate in 2009, the first woman elected to the Senate was Hattie Wyatt Caraway of Arkansas. She was appointed to fill the vacancy caused by the death of her husband, then won election in 1932, was reelected in 1938, and served until 1945. Table 1.43 shows the number of female senators at the beginning of the first session of each Congress.<sup>25</sup>
  - (a) Is the number of female senators a function of the Congress's number,  $c$ ? Explain.
  - (b) Is the Congress's number a function of the number of female senators? Explain.
  - (c) Let  $S(c)$  represent the number of female senators serving in the  $c^{\text{th}}$  Congress. What does the statement  $S(104) = 8$  mean?
  - (d) Evaluate and interpret  $S(110)$ .

Table 1.43 Female senators,  $S$ , in Congress  $c$ 

$c$	96	98	100	102	104	106	108	110	111
$S$	1	2	2	2	8	9	14	16	17

31. A light is turned off for several hours. It is then turned on. After a few hours it is turned off again. Sketch the light bulb's temperature as a function of time.
32. According to Charles Osgood, CBS news commentator, it takes about one minute to read 15 double-spaced typewritten lines on the air.<sup>26</sup>
  - (a) Construct a table showing the time Charles Osgood is reading on the air in seconds as a function of the number of double-spaced lines read for 0, 1, 2,  $\dots$ , 10 lines. From your table, how long does it take Charles Osgood to read 9 lines?
  - (b) Plot this data on a graph with the number of lines on the horizontal axis.
  - (c) From your graph, estimate how long it takes Charles Osgood to read 9 lines. Estimate how many lines Charles Osgood can read in 30 seconds.
  - (d) Construct a formula which relates the time  $T$  to  $n$ , the number of lines read.
33. The distance between Cambridge and Wellesley is 10 miles. A person walks part of the way at 5 miles per hour, then jogs the rest of the way at 8 mph. Find a formula that expresses the total amount of time for the trip,  $T(d)$ , as a function of  $d$ , the distance walked.
34. A cylindrical can is closed at both ends and its height is twice its radius. Express its surface area,  $S$ , as a function of its radius,  $r$ . [Hint: The surface of a can consists of a rectangle plus two circular disks.]
35. A lawyer does nothing but sleep and work during a day. There are 1440 minutes in a day. Write a linear function relating minutes of sleep,  $s$ , to minutes of work,  $w$ .

For the functions in Problems 36–38:

- (a) Find the average rate of change between the points
    - (i)  $(-1, f(-1))$  and  $(3, f(3))$
    - (ii)  $(a, f(a))$  and  $(b, f(b))$
    - (iii)  $(x, f(x))$  and  $(x + h, f(x + h))$
  - (b) What pattern do you see in the average rate of change between the three pairs of points?
36.  $f(x) = 5x - 4$
  37.  $f(x) = \frac{1}{2}x + \frac{5}{2}$
  38.  $f(x) = x^2 + 1$

<sup>24</sup>Data from the National Hurricane Center, [www.nhc.noaa.gov/aboutwindprofile.shtml](http://www.nhc.noaa.gov/aboutwindprofile.shtml), accessed October 7, 2004.

<sup>25</sup>[http://en.wikipedia.org/wiki/111th\\_United\\_States\\_Congress\\_Members](http://en.wikipedia.org/wiki/111th_United_States_Congress_Members).

<sup>26</sup>T. Parker, *Rules of Thumb* (Boston: Houghton Mifflin, 1983).

39. Table 1.44 gives the average temperature,  $T$ , at a depth  $d$ , in a borehole in Belleterre, Quebec.<sup>27</sup> Evaluate  $\Delta T/\Delta d$  on the following intervals, and explain what your answers tell you about borehole temperature.

- (a)  $25 \leq d \leq 150$   
 (b)  $25 \leq d \leq 75$   
 (c)  $100 \leq d \leq 200$

Table 1.44

$d$ , depth (m)	25	50	75	100
$T$ , temp ( $^{\circ}\text{C}$ )	5.50	5.20	5.10	5.10
$d$ , depth (m)	125	150	175	200
$T$ , temp ( $^{\circ}\text{C}$ )	5.30	5.50	5.75	6.00
$d$ , depth (m)	225	250	275	300
$T$ , temp ( $^{\circ}\text{C}$ )	6.25	6.50	6.75	7.00

40. The population,  $P(t)$ , in millions, of a country in year  $t$ , is given by the formula  $P(t) = 22 + 0.3t$ .

- (a) Construct a table of values for  $t = 0, 10, 20, \dots, 50$ .  
 (b) Plot the points you found in part (a).  
 (c) What is the country's initial population?  
 (d) What is the average rate of change of the population, in millions of people/year?

41. A woodworker sells rocking horses. His start-up costs, including tools, plans, and advertising, total \$5000. Labor and materials for each horse cost \$350.

- (a) Calculate the woodworker's total cost,  $C$ , to make 1, 2, 5, 10, and 20 rocking horses. Graph  $C$  against  $n$ , the number of rocking horses that he carves.  
 (b) Find a formula for  $C$  in terms of  $n$ .  
 (c) What is the rate of change of the function  $C$ ? What does the rate of change tell us about the woodworker's expenses?

42. Outside the US, temperature readings are usually given in degrees Celsius; inside the US, they are often given in degrees Fahrenheit. The exact conversion from Celsius,  $C$ , to Fahrenheit,  $F$ , uses the formula

$$F = \frac{9}{5}C + 32.$$

An approximate conversion is obtained by doubling the temperature in Celsius and adding  $30^{\circ}$  to get the equivalent Fahrenheit temperature.

- (a) Write a formula using  $C$  and  $F$  to express the approximate conversion.

- (b) How far off is the approximation if the Celsius temperature is  $-5^{\circ}, 0^{\circ}, 15^{\circ}, 30^{\circ}$ ?  
 (c) For what temperature (in Celsius) does the approximation agree with the actual formula?

43. Find a formula for the linear function  $h(t)$  whose graph intersects the graph of  $j(t) = 30(0.2)^t$  at  $t = -2$  and  $t = 1$ .

44. Find the equation of the line  $l$  in Figure 1.68. The shapes under the line are squares.

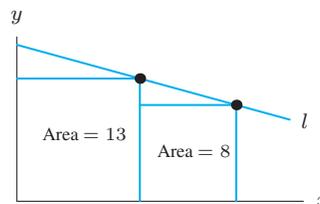


Figure 1.68

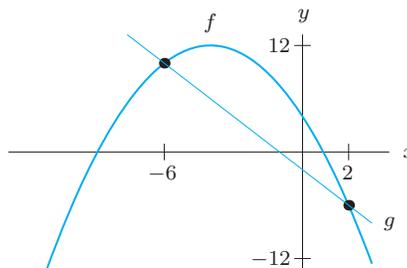
45. A bullet is shot straight up into the air from ground level. After  $t$  seconds, the velocity of the bullet, in meters per second, is approximated by the formula

$$v = f(t) = 1000 - 9.8t.$$

- (a) Evaluate the following:  $f(0), f(1), f(2), f(3), f(4)$ . Compile your results in a table.  
 (b) Describe in words what is happening to the speed of the bullet. Discuss why you think this is happening.  
 (c) Evaluate and interpret the slope and both intercepts of  $f(t)$ .  
 (d) The gravitational field near the surface of Jupiter is stronger than that near the surface of the earth, which, in turn, is stronger than the field near the surface of the moon. How is the formula for  $f(t)$  different for a bullet shot from Jupiter's surface? From the moon?  
 46. A theater manager graphed weekly profits as a function of the number of patrons and found that the relationship was linear. One week the profit was \$11,328 when 1324 patrons attended. Another week 1529 patrons produced a profit of \$13,275.50.  
 (a) Find a formula for weekly profit,  $y$ , as a function of the number of patrons,  $x$ .  
 (b) Interpret the slope and the  $y$ -intercept.  
 (c) What is the break-even point (the number of patrons for which there is zero profit)?

<sup>27</sup>Hugo Beltrami of St. Francis Xavier University and David Chapman of the University of Utah posted this data at <http://geophysics.stfx.ca/public/borehole/borehole.html>, accessed November 10, 2005.

- (d) Find a formula for the number of patrons as a function of profit.
- (e) If the weekly profit was \$17,759.50, how many patrons attended the theater?
47. Describe a linear (or nearly linear) relationship that you have encountered outside the classroom. Determine the rate of change and interpret it in practical terms.
48. In economics, the *demand* for a product is the amount of that product that consumers are willing to buy at a given price. The quantity demanded of a product usually decreases if the price of that product increases. Suppose that a company believes there is a linear relationship between the demand for its product and its price. The company knows that when the price of its product was \$3 per unit, the quantity demanded weekly was 500 units, and that when the unit price was raised to \$4, the quantity demanded weekly dropped to 300 units. Let  $D$  represent the quantity demanded weekly at a unit price of  $p$  dollars.
- Calculate  $D$  when  $p = 5$ . Interpret your result.
  - Find a formula for  $D$  in terms of  $p$ .
  - The company raises the price of the good and that the new quantity demanded weekly is 50 units. What is the new price?
  - Give an economic interpretation of the slope of the function you found in part (b).
  - Find  $D$  when  $p = 0$ . Find  $p$  when  $D = 0$ . Give economic interpretations of both these results.
49. In economics, the *supply* of a product is the quantity of that product suppliers are willing to provide at a given price. In theory, the quantity supplied of a product increases if the price of that product increases. Suppose that there is a linear relationship between the quantity supplied,  $S$ , of the product described in Problem 48 and its price,  $p$ . The quantity supplied weekly is 100 when the price is \$2 and the quantity supplied rises by 50 units when the price rises by \$0.50.
- Find a formula for  $S$  in terms of  $p$ .
  - Interpret the slope of your formula in economic terms.
  - Is there a price below which suppliers will not provide this product?
  - The *market clearing price* is the price at which supply equals demand. According to theory, the free-market price of a product is its market clearing price. Using the demand function from Problem 48, find the market clearing price for this product.
50. When economists graph demand or supply equations, they place quantity on the horizontal axis and price on the vertical axis.
- On the same set of axes, graph the demand and supply equations you found in Problems 48 and 49, with price on the vertical axis.
  - Indicate how you could estimate the market clearing price from your graph.
51. The figure gives graphs of  $g$ , a linear function, and of  $f(x) = 12 - 0.5(x + 4)^2$ . Find a possible formula for  $g$ .



52. Write in slope-intercept form and identify the values of  $b$  and  $m$ :

$$f(r) = rx^3 + 3rx^2 + 2r + 4sx + 7s + 3.$$

53. Find an equation for the line intersecting the graph of  $f$  at  $x = -2$  and  $x = 5$  given that  $f(x) = 2 + \frac{3}{x+5}$ .
54. A business consultant works 10 hours a day, 6 days a week. She divides her time between meetings with clients and meetings with co-workers. A client meeting requires 3 hours while a co-worker meeting requires 2 hours. Let  $x$  be the number of co-worker meetings the consultant holds during a given week. If  $y$  is the number of client meetings for which she has time remaining, then  $y$  is a function of  $x$ . Assume this relationship is linear and that meetings can be split up and continued on different days.
- Graph the relationship between  $y$  and  $x$ . [Hint: Consider the maximum number of client and co-worker meetings that can be held.]
  - Find a formula for  $y$  as a function of  $x$ .
  - Explain what the slope and the  $x$ - and  $y$ -intercepts represent in the context of the consultant's meeting schedule.
  - A change is made so that co-worker meetings take 90 minutes instead of 2 hours. Graph this situation. Describe those features of this graph that have changed from the one sketched in part (a) and those that have remained the same.
55. You start 60 miles east of Pittsburgh and drive east at a constant speed of 50 miles per hour. (Assume that the road is straight and permits you to do this.) Find a formula for  $d$ , your distance from Pittsburgh as a function of  $t$ , the number of hours of travel.

56. Find a formula for the line parallel to the line  $y = 20 - 4x$  and containing the point  $(3, 12)$ .
57. Find the equation of the linear function  $g$  whose graph is perpendicular to the line  $5x - 3y = 6$ ; the two lines intersect at  $x = 15$ .
58. Find the coordinates of point  $P$  in Figure 1.69.

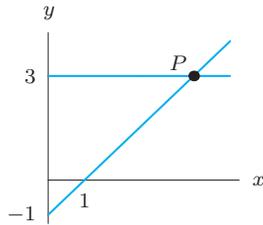


Figure 1.69

59. You want to choose one long-distance telephone company from the following options.
- Company A charges \$0.37 per minute.
  - Company B charges \$13.95 per month plus \$0.22 per minute.
  - Company C charges a fixed rate of \$50 per month.

Let  $Y_A$ ,  $Y_B$ ,  $Y_C$  represent the monthly charges using Company A, B, and C, respectively. Let  $x$  be the number of minutes per month spent on long-distance calls.

- (a) Find formulas for  $Y_A$ ,  $Y_B$ ,  $Y_C$  as functions of  $x$ .
- (b) Figure 1.70 gives the graphs of the functions in part (a). Which function corresponds to which graph?
- (c) Find the  $x$ -values for which Company B is cheapest.

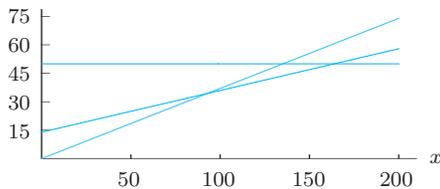


Figure 1.70

60. A commission is a payment made to an employee based on a percentage of sales made. For example, car salespeople earn commission on the selling price of a car. In parts (a)–(d), explain how to choose between the options for different levels of sales.
- (a) A weekly salary of \$100 or a weekly salary of \$50 plus 10% commission.

- (b) A weekly salary of \$175 plus 7% commission or a weekly salary of \$175 plus 8% commission.
- (c) A weekly salary of \$145 plus 7% commission or a weekly salary of \$165 plus 7% commission.
- (d) A weekly salary of \$225 plus 3% commission or a weekly salary of \$180 plus 6% commission.

61. Table 1.45 shows the IQ of ten students and the number of hours of TV each watches per week.

- (a) Make a scatter plot of the data.
- (b) By eye, make a rough estimate of the correlation coefficient.
- (c) Use a calculator or computer to find the least squares regression line and the correlation coefficient. Your values should be correct to four decimal places.

Table 1.45

IQ	110	105	120	140	100	125	130	105	115	110
TV	10	12	8	2	12	10	5	6	13	3

62. For 35 years, major league baseball Hall of Fame member Henry Aaron held the record for the greatest number of career home runs. His record was broken by Barry Bonds in 2007. Table 1.46 shows Aaron's cumulative yearly record<sup>28</sup> from the start of his career, 1954, until 1973.

- (a) Plot Aaron's cumulative number of home runs  $H$  on the vertical axis, and the time  $t$  in years along the horizontal axis, where  $t = 1$  corresponds to 1954.
- (b) By eye, draw a straight line that fits these data well and find its equation.
- (c) Use a calculator or computer to find the equation of the regression line for these data. What is the correlation coefficient,  $r$ , to 4 decimal places? To 3 decimal places? What does this tell you?
- (d) What does the slope of the regression line mean in terms of Henry Aaron's home-run record?
- (e) From your answer to part (d), how many home runs do you estimate Henry Aaron hit in each of the years 1974, 1975, 1976, and 1977? If you were told that Henry Aaron retired at the end of the 1976 season, would this affect your answers?

Table 1.46 Henry Aaron's cumulative home-run record,  $H$ , from 1954 to 1973, with  $t$  in years since 1953

$t$	1	2	3	4	5	6	7	8	9	10
$H$	13	40	66	110	140	179	219	253	298	342
$t$	11	12	13	14	15	16	17	18	19	20
$H$	366	398	442	481	510	554	592	639	673	713

<sup>28</sup>Adapted from "Graphing Henry Aaron's home-run output" by H. Ringel, *The Physics Teacher*, January 1974, page 43.

63. The graph of a linear function  $y = f(x)$  passes through the two points  $(a, f(a))$  and  $(b, f(b))$ , where  $a < b$  and  $f(a) < f(b)$ .

- (a) Graph the function labeling the two points.  
 (b) Find the slope of the line in terms of  $f$ ,  $a$ , and  $b$ .

64. Let  $f(x) = 0.003 - (1.246x + 0.37)$ .

(a) Calculate the following average rates of change:

(i)  $\frac{f(2) - f(1)}{2 - 1}$       (ii)  $\frac{f(1) - f(2)}{1 - 2}$

(iii)  $\frac{f(3) - f(4)}{3 - 4}$

(b) Rewrite  $f(x)$  in the form  $f(x) = b + mx$ .

Write the linear function  $y = -3 - x/2$  in the forms given in Problems 65–66, assuming all constants are positive.

65.  $y = \frac{p}{p-1} - r^2x$       66.  $y = \frac{x+k}{z}$

67. You spend  $c$  dollars on  $x$  apples and  $y$  bananas. In Figure 1.71, line  $l$  gives  $y$  as a function of  $x$ .

- (a) If apples cost  $p$  dollars each and bananas cost  $q$  each, label the  $x$ - and  $y$ -intercepts of  $l$ . [Note: Your labels will involve the constants  $p$ ,  $q$  or  $c$ .]  
 (b) What is the slope of  $l$ ?

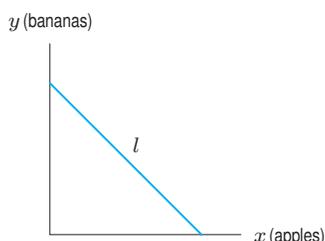


Figure 1.71: Axes not necessarily to scale

68. The apples in Problem 67 cost more than bananas, so  $p > q$ . Which of the two lines,  $l_1$  or  $l_2$ , in Figure 1.72 could represent  $y = f(x)$ ?

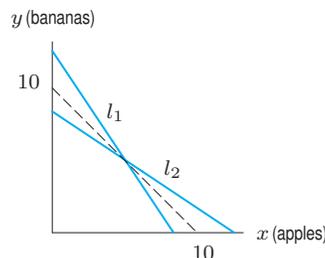


Figure 1.72

69. Many people think that hair growth is stimulated by haircuts. In fact, there is no difference in the rate hair grows after a haircut, but there *is* a difference in the rate at which hair's ends break off. A haircut eliminates dead and split ends, thereby slowing the rate at which hair breaks. However, even with regular haircuts, hair will not grow to an indefinite length. The average life cycle of human scalp hair is 3–5 years, after which the hair is shed.<sup>29</sup>

Judy trims her hair once a year, when its growth is slowed by split ends. She cuts off just enough to eliminate dead and split ends, and then lets it grow another year. After 5 years, she realizes her hair won't grow any longer. Graph the length of her hair as a function of time. Indicate when she receives her haircuts.

70. Academics have suggested that loss of worker productivity can result from sleep deprivation. An article in the September 26, 1993, *New York Times* quotes David Poltrack, the senior vice president for planning and research at CBS, as saying that seven million Americans are staying up an hour later than usual to watch talk show host David Letterman. The article goes on to quote Timothy Monk, a professor at the University of Pittsburgh School of Medicine, as saying, "... my hunch is that the effect [on productivity due to sleep deprivation among this group] would be in the area of a 10 percent decrement." The article next quotes Robert Solow, a Nobel prize-winning professor of economics at MIT, who suggests the following procedure to estimate the impact that this loss in productivity will have on the US economy—an impact he dubbed "the Letterman loss." First, Solow says, we find the percentage of the work force who watch the program. Next, we determine this group's contribution to the gross domestic product (GDP). Then we reduce the group's contribution by 10% to account for the loss in productivity due to sleep deprivation. The amount of this reduction is "the Letterman loss."

- (a) The article estimated that the GDP is \$6.325 trillion, and that 7 million Americans watch the show. Assume that the nation's work force is 118 million

<sup>29</sup>*Britannica Micropaedia* vol. 5 (Chicago: Encyclopaedia Britannica, Inc., 1989).