CHAPTER ONE

Solutions for Section 1.1

Skill Refresher

S1. Finding the common denominator we get $c + \frac{1}{2}c = \frac{2c+c}{2} = \frac{3c}{2} = \frac{3}{2}c$.

S2. $P(1 + 0.07 + 0.02) = 1.09P$.

S3. $2\pi r^2 + 2\pi r \cdot 2r = 2\pi r^2 + 4\pi r^2 = 6\pi r^2$.

S4. $\dfrac{12\pi - 2\pi}{6\pi} = \dfrac{10\pi}{6\pi} = \dfrac{5}{3}$.

S5. $\left(\frac{1}{2}\right) - 5(-5) = \frac{1}{2} + 25 = \frac{51}{2}$.

S6. $1 - 12(3) + (3)^2 = 1 - 36 + 9 = -26$.

S7. $\dfrac{3}{2-(-1)^3} = \dfrac{3}{2-(-1)} = \dfrac{3}{1+1} = \dfrac{3}{2}$.

S8. $\dfrac{4}{1+\frac{1}{-\frac{3}{4}}} = \dfrac{4}{1-\frac{4}{3}} = \dfrac{4}{-\frac{1}{3}} = -12$.

S9. The figure is a parallelogram, so $A = (-2, 8)$.

S10. The figure is a parallelogram, so $A = (3, 21)$.

Exercises

1. **(a)** On the graph, the high tides occur when the graph is at its highest points. On this particular day, there were two high tides.

(b) The low tides occur when the graph is at its lowest points. There were two low tides on this day.

(c) To find the amount of time elapsed between high tides, find the distance between the two highest points on the graph. It is about 12 hours.

2. $m = f(v)$.

3. $w = f(c)$.

4. Appropriate axes are shown in Figure 1.1.

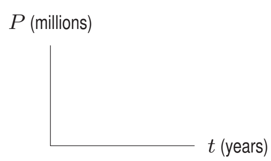

Figure 1.1

5. Appropriate axes are shown in Figure 1.2.

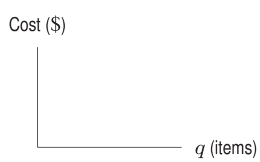

Figure 1.2

6. Appropriate axes are shown in Figure 1.3.

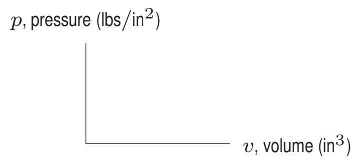

Figure 1.3

7. Appropriate axes are shown in Figure 1.4.

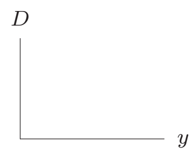

Figure 1.4

8. These data are plotted in Figure 1.5. The independent variable is A and the dependent variable is n.

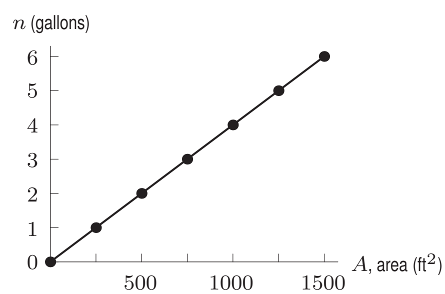

Figure 1.5

9. **(a)** Since $f(x)$ is 4 when $x = 0$, we have $f(0) = 4$.
(b) Since $x = 3$ when $f(x) = 0$, we have $f(3) = 0$.
(c) $f(1) = 2$
(d) There are two x values leading to $f(x) = 1$, namely $x = 2$ and $x = 4$. So $f(2) = 1$ and $f(4) = 1$.

10. **(a)** Since the vertical intercept is $(0, 40)$, we have $f(0) = 40$.
(b) Since the horizontal intercept is $(2, 0)$, we have $f(2) = 0$.

11. $f(6.9) = 2.9$.

12. $(2.2, 2.9)$; $(6.1, 4.9)$

13. Since $f(0) = f(4) = f(8) = 0$, the solutions are $x = 0, 4, 8$.

14. Since the graphs touch at $x = 2.2$ and $x = 6.1$, these are the solutions.

15. **(a)** w goes on the horizontal axis
(b) $(-4, 10)$
(c) $(6, 1)$

16. **(a)** No. Because there can be two different points sharing the same x-coordinate. For example, when $x = 0$, $y = 1$ or $y = -1$. So for each value of x, there is not a unique value of y.

(b) Yes. Because on the semi-circle above the x-axis there is only one point for each x-coordinate. Thus, each x-value corresponds to at most one y-value.

17. **(a)** Yes. For each value of s, there is exactly one area.

(b) No. Suppose $s = 4$ represents the length of the rectangle. The width could have any other value, say 7 or 1.5 or π or . . .. In this case, for one value of s, there are infinitely many possible values for A, so the area of a rectangle is not a function of the length of one of its sides.

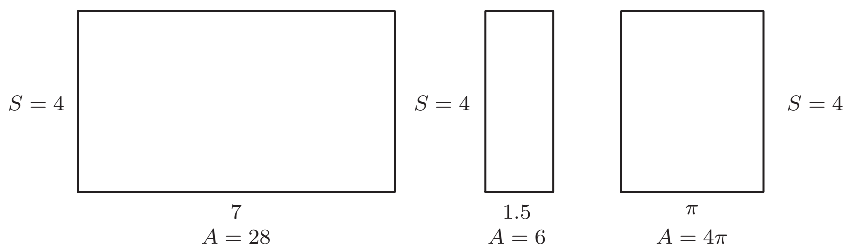

Figure 1.6

18. We apply the vertical-line test to each graph. As you can see, only in (a) do all vertical lines intersect only one point on the graph. So graph (a) defines the only function.

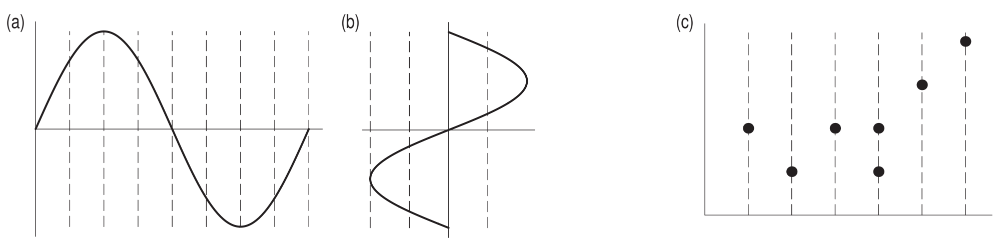

Figure 1.7

Problems

19. A possible graph is shown in Figure 1.8

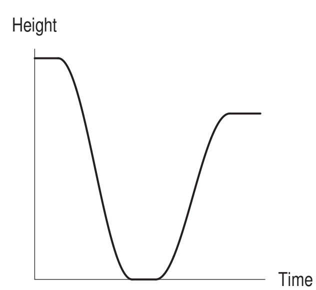

Figure 1.8

20. Figure 1.9 shows a possible graph of blood sugar level as a function of time over one day. Note that the actual curve is smooth, and does not have any sharp corners.

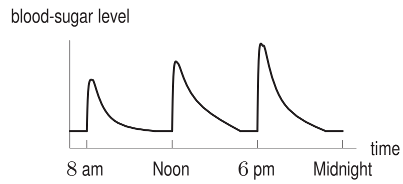

Figure 1.9

21. **(a)** The number of people who own cell phones in the year 2000 is 100,300,000.
(b) There are 20,000,000 people who own cell phones a years after 1990.
(c) There will be b million people who own cell phones in the year 2010.
(d) The number n is the number of people (in millions) who own cell phones t years after 1990.

22. The value of N is not necessarily a function of G, since each value of G does not need to have a unique value of N associated to it. For example, suppose we choose the value of G to be a B. There may be more than one student who received a B, so there may be more than one ID number corresponding to B.

The value of G must be a function of N, because each ID number (each student) receives exactly one grade. Therefore each value of N has a unique value of G associated with it. Writing $G = f(N)$ indicates that the ID number is the input which uniquely determines the grade, the output.

23. **(a)** In 1995, or year $t = 0$, the ranking for Hannah was 7, making it most popular, and the ranking for Madison was 29, making it least popular.
(b) In 2004, or year $t = 9$, the ranking for Madison was 3, making it the most popular, and the ranking for Alexis was 11, making it least popular.

24. **(a)** We have $r_m(0) - r_h(0) = 29 - 7 = 22$. This tells us that in 1995 (year $t = 0$), the name Hannah was ranked 22 places higher than Madison on the list of most popular names. (Recall that the lower the ranking, the higher a name's position on the list.)
(b) We have $r_m(9) - r_h(9) = 3 - 5 = -2$. This tells us that in 2004 (year $t = 9$), the name Hannah was ranked 2 places lower than Madison on the list of most popular names.
(c) We have $r_m(t) < r_a(t)$ for $t = 5$ to $t = 9$. This tells us that the name Madison was ranked higher than the name Alexis on the list of most popular names in the years 2000 to 2004.

25. **(a)** At 40 mph, fuel consumption is about 28 mpg, so the fuel used is $300/28 = 10.71$ gallons.
(b) At 60 mph, fuel consumption is about 29 mpg. At 70 mph, fuel consumption is about 28 mpg. Therefore, on a 200 mile trip

$$\text{Fuel saved } = \frac{200}{28} - \frac{200}{29} = 0.25 \text{ gallons.}$$

(c) The most fuel-efficient speed is where mpg is a maximum, which is about 55 mph.

26. **(a)** When there is no snow, it is equivalent to no rain. Thus, the vertical intercept is 0. Since every ten inches of snow is equivalent to one inch of rain, we can specify the slope as

$$\frac{\Delta \text{rain}}{\Delta \text{snow}} = \frac{1}{10} = 0.1$$

We have a vertical intercept of 0 and a slope of 0.1. Thus, the equation is: $r = f(s) = 0.1s$.
(b) By substituting 5 in for s, we get $f(5) = 0.1(5) = 0.5$ This tells us that five inches of snow is equivalent to approximately $1/2$ inch of rain.
(c) Substitute 5 inches for $r = f(s)$ in the equation: $5 = 0.1s$. Solving gives $s = 50$. Five inches of rain is equivalent to approximately 50 inches of snow.

27. Figure 1.10 shows the tank.

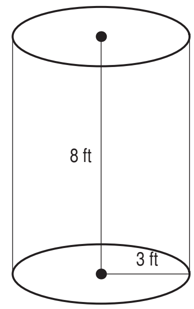

Figure 1.10: Cylindrical water tank

(a) The volume of a cylinder is equal to the area of the base times the height, where the area of the base is πr^2. Here, the radius of the base is $(1/2)(6) = 3$ ft, so the area is $\pi \cdot 3^2 = 9\pi$ ft^2. Therefore, the capacity of this tank is $(9\pi)8 = 72\pi$ ft^3.

(b) If the height of the water is 5 ft, the volume becomes $(9\pi)5 = 45\pi$ ft^3.

(c) In general, if the height of water is h ft, the volume of the water is $(9\pi)h$. If we let $V(h)$ be the volume of water in the tank as a function of its height, then

$$V(h) = 9\pi h.$$

Note that this function only makes sense for a non-negative value of h, which does not exceed 8 feet, the height of the tank.

28. **(a)** Since the person starts out 5 miles from home, the vertical intercept on the graph must be 5. Thus, (i) and (ii) are possibilities. However, since the person rides 5 mph away from home, after 1 hour the person is 10 miles from home. Thus, (ii) is the correct graph.

(b) Since this person also starts out 5 miles from home, (i) and (ii) are again possibilities. This time, however, the person is moving at 10 mph and so is 15 miles from home after 1 hour. Thus, (i) is correct.

(c) The person starts out 10 miles from home so the vertical intercept must be 10. The fact that the person reaches home after 1 hour means that the horizontal intercept is 1. Thus, (v) is correct.

(d) Starting out 10 miles from home means that the vertical intercept is 10. Being half way home after 1 hour means that the distance from home is 5 miles after 1 hour. Thus, (iv) is correct.

(e) We are looking for a graph with vertical intercept of 5 and where the distance is 10 after 1 hour. This is graph (ii). Notice that graph (iii), which depicts a bicyclist stopped 10 miles from home, does not match any of the stories.

29. **(a)** 69°F

(b) July 17^{th} and 20^{th}

(c) Yes. For each date, there is exactly one low temperature.

(d) No, it is not true that for each low temperature, there is exactly one date: for example, 73° corresponds to both the 17^{th} and 20^{th}.

30. **(a)** Figure 1.11 shows the plot of R versus t. R is a function of t because no vertical line intersects the graph in more than one place.

(b) Figure 1.12 shows the plot of F versus t. F is a function of t because no vertical line intersects the graph in more than one place.

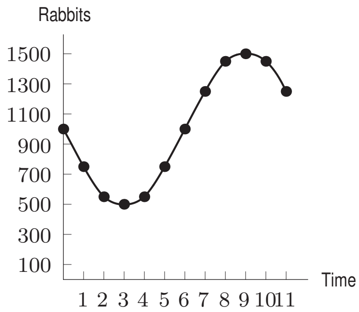

Figure 1.11: The graph of R versus t

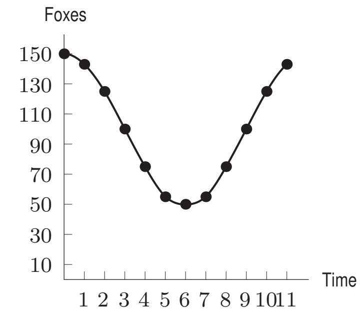

Figure 1.12: The graph of F versus t

(c) Figure 1.13 shows the plot of F versus R. We have also drawn the vertical line corresponding to $R = 567$. This tells us that F is not a function of R because there is a vertical line that intersects the graph twice. In fact the lines $R = 567$, $R = 750$, $R = 1000$, $R = 1250$, and $R = 1433$ all intersect the graph twice. However, the existence of any one of them is enough to guarantee that F is not a function of R.

(d) Figure 1.14 shows the plot of R versus F. We have drawn the vertical line corresponding to $F = 57$. This tells us that R is not a function of F because there is a vertical line that intersects the graph twice. In fact the lines $F = 57$, $F = 75$, $F = 100$, $F = 125$, and $F = 143$ all intersect the graph twice. However, the existence of any one of them is enough to guarantee that R is not a function of F.

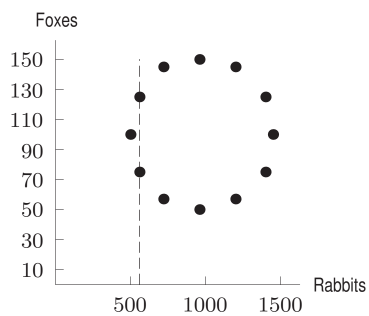

Figure 1.13: The graph of F versus R

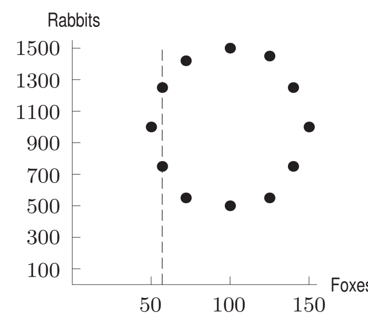

Figure 1.14: The graph of R versus F

31. **(a)** No, in the year 1954 there were two world records; in the year 1981 there were three world records.
(b) Yes, each world record occurred in only one year.
(c) The world record of 3 minutes and 47.33 seconds was set in 1981.
(d) The statement $y(3{:}51.1) = 1967$ tells us that the world record of 3 minutes, 51.1 seconds was set in 1967.

32. **(a)** From the table, we see that $f(100) = 625$. This means that there is approximately \$625 billion worth of \$100 bills in circulation in the United States.
(b) To determine the number of \$5 bills, we divide 11 by 5. Thus, we have 2.2 billion \$5 bills in circulation. The number of \$1 bills is the same as the value, so there are 9.5 billion \$1 bills in circulation. There are more \$1 bills.

33. **(a)** Adding the male total to the female total gives $x + y$, the total number of applicants.
(b) Of the men who apply, 15% are accepted. So $0.15x$ male applicants are accepted. Likewise, 18% of the women are accepted so we have $0.18y$ women accepted. Summing the two tells us that $0.15x + 0.18y$ applicants are accepted.
(c) The number accepted divided by the number who applied times 100 gives the percentage accepted. This expression is

$$\frac{(0.15)x + (0.18)y}{x + y}(100), \quad \text{or} \quad \frac{15x + 18y}{x + y}.$$

34. Since the tax is $0.06P$, the total cost would be the price of the item plus the tax,or

$$C = P + 0.06P = 1.06P.$$

35. Let $A(r)$ be the area of a circle expressed as a function of its radius. Then

$$A(r) = \pi r^2.$$

If the radius is increased by 10%, then it is 110% of its original length. So we want to know the output when our input is $1.1r$:

$$A(1.1r) = \pi(1.1r)^2 = 1.21\pi r^2.$$

The new area is the old area multiplied by 1.21. So the new area is 121% of the old area. In other words, the area of a circle is increased by 21% when its radius is increased by 10%.

36. The original price is P. Inflation causes a 5% increase, giving

$$\text{Inflated price} = P + 0.05P = 1.05P.$$

Then there is a 10% decrease, giving

$$\begin{aligned}\text{Final price} &= 90\%(\text{Inflated price})\\ &= 0.9(1.05P)\\ &= 0.945P.\end{aligned}$$

37. (a)

Table 1.1 *Relationship between cost, C, and number of liters produced, l*

l (millions of liters)	0	1	2	3	4	5
C (millions of dollars)	2.0	2.5	3.0	3.5	4.0	4.5

(b) The cost, C, consists of a fixed cost of \$2 million plus a variable cost of \$0.50 million per million liters produced. If l millions of liters are produced, the total variable costs are $(0.5)l$. Thus, the total cost C in millions of dollars is given by

$$C = \text{Fixed cost} + \text{Variable cost},$$

so

$$C = 2 + (0.5)l.$$

38. (a) Yes. If the person walks due west and then due north, the distance from home is represented by the hypotenuse of the right triangle that is formed (see Figure 1.15).

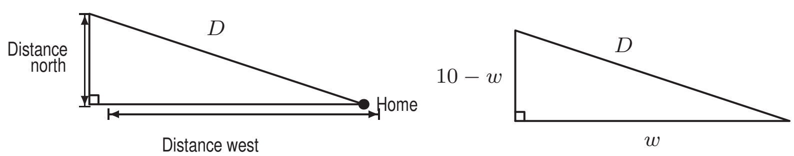

Figure 1.15

If the distance west is w miles and the total distance walked is 10 miles, then the distance north is $10 - w$ miles. We can use the Pythagorean Theorem to find that

$$D = \sqrt{w^2 + (10 - w)^2}.$$

So, for each value of w, there is a unique value of D given by this formula. Thus, the definition of a function is satisfied.