Topics 9.7 - 9.9 Unit 9b Review

Part I. Multiple Choice

Circle the best answer for each problem. You may not use a calculator.

1. Which of the following set of parametric equations would graph the polar curve with the equation $r = 2 - 3\cos\theta$?

(A) $x = 2\sin\theta - 3\sin\theta\cos\theta$, $y = 2\cos\theta - 3\cos^2\theta$

(B) $x = 3\cos^2\theta - 2\cos\theta$, $y = 3\sin\theta\cos\theta - 2\sin\theta$

(C) $x = 2\cos\theta - 3\cos^2\theta$, $y = 2\sin\theta - 3\sin\theta\cos\theta$

(D) $x = 2\sin\theta - 6\cos\theta\sin\theta$, $y = 2\cos\theta - 3\left(\cos^2\theta - \sin^2\theta\right)$

2. Suppose the function $r = f(\theta)$ is differentiable. The table gives values for f and f' at $\theta = \frac{\pi}{2}$.

θ	$f(\theta)$	$f'(\theta)$
$\frac{\pi}{2}$	10	4

What is the slope of the line tangent to the graph of $r = f(\theta)$ at the point where $\theta = \frac{\pi}{2}$?

(A) $\frac{2}{5}$ **(B)** $-\frac{2}{5}$ **(C)** $\frac{4}{5}$ **(D)** $-\frac{4}{5}$

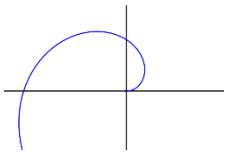
3. What is the slope of the tangent line to the cardioid $r = 1 + \cos\theta$ at $\theta = \frac{\pi}{6}$?

(A) -1 **(B)** $-\frac{\sqrt{3}+1}{2}$ **(C)** $\frac{\sqrt{3}+1}{2}$ **(D)** 1

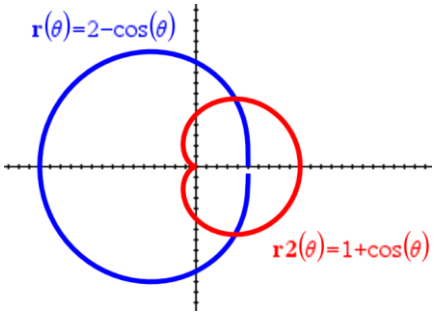
4. Find the area of the region enclosed by the polar curve $r = \theta$ and the horizontal axis as shown in the figure.

(A) $\frac{\pi^2}{4}$ **(B)** $\frac{\pi^3}{6}$

(C) $\frac{\pi^3}{3}$ **(D)** $\frac{\pi^3}{2}$



5. Let R be the region that lies inside the polar curves $r = 2 - \cos\theta$ and inside the polar curve $r = 1 + \cos\theta$. Which of the following correctly sets up the area of this region?



(A) $A = 2\left[\frac{1}{2}\int_0^{\pi/3}(2^2 - \cos^2\theta)\,d\theta + \frac{1}{2}\int_{\pi/3}^{\pi}(1 + \cos^2\theta)\,d\theta\right]$

(B) $A = 2\left[\frac{1}{2}\int_0^{\pi/4}(2 - \cos\theta)^2\,d\theta + \frac{1}{2}\int_{\pi/4}^{\pi}(1 + \cos\theta)^2\,d\theta\right]$

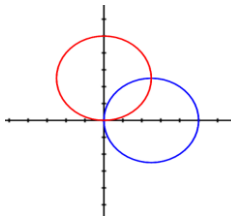
(C) $A = 2\left[\frac{1}{2}\int_0^{\pi/3}(2 - \cos\theta)^2\,d\theta + \frac{1}{2}\int_{\pi/3}^{\pi}(1 + \cos\theta)^2\,d\theta\right]$

(D) $A = 2\left[\frac{1}{2}\int_0^{\pi/3}(2 - \cos\theta)^2\,d\theta - \frac{1}{2}\int_{\pi/3}^{\pi}(1 + \cos\theta)^2\,d\theta\right]$

6. Find the area of the region that lies inside both the curve $r = cos\,\theta$ and $r = sin\,\theta$.

(A) $\frac{1}{8}(\pi - 2)$ **(B)** $\frac{1}{4}(\pi - 2)$

(C) $\frac{1}{2}(\pi - 2)$ **(D)** $\frac{1}{8}(\pi - 1)$



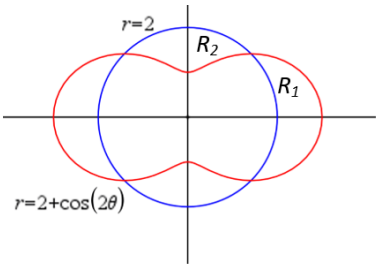
Part II. Free Response

7. Consider the polar equation $r = 2 - 2\,cos\,\theta$.

a. Calculate the derivative, $\frac{dy}{dx}$, for the given polar equation. You do not have to simplify.

b. Find all values of θ for which the polar graph has only a <u>vertical</u> tangent line.

8. The figure to the right shows the graphs of the polar curves $r = 2$ and $r = 2 + cos(2\theta)$. Let R_1 be the shaded region in the first quadrant bounded by the two curves and the horizontal axis, and R_2 be the shaded region in the first quadrant bounded by the two curves and the vertical axis.



a. Set up, but do not evaluate, an integral expression that represent the area of region R_1.

b. Set up, but do not evaluate, an integral expression that represents the area of region R_2.