Chapter 9

Section 9.1 *(page 625)*

1. 7, 9, 11, 13, 15 **3.** 2, 4, 8, 16, 32

5. $-2, 4, -8, 16, -32$ **7.** $2, \frac{3}{2}, \frac{4}{3}, \frac{5}{4}, \frac{6}{5}$

9. $3, \frac{12}{11}, \frac{9}{13}, \frac{24}{47}, \frac{15}{37}$ **11.** $0, 1, 0, \frac{1}{2}, 0$

13. $\frac{5}{2}, \frac{11}{4}, \frac{23}{8}, \frac{47}{16}, \frac{95}{32}$ **15.** $1, \frac{1}{2^{3/2}}, \frac{1}{3^{3/2}}, \frac{1}{4^{3/2}}, \frac{1}{5^{3/2}}$

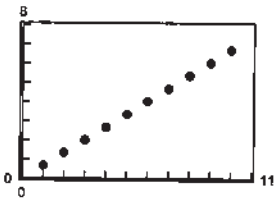
17. $3, \frac{9}{2}, \frac{9}{2}, \frac{27}{8}, \frac{81}{40}$ **19.** $-1, \frac{1}{4}, -\frac{1}{9}, \frac{1}{16}, -\frac{1}{25}$

21. 3, 15, 35, 63, 99 **23.** -73 **25.** $\frac{4}{14{,}175}$

27. $\frac{48}{285} = \frac{16}{95}$ **29.** 28, 24, 20, 16, 12

31. 3, 4, 6, 10, 18 **33.** 2, 6, 10, 22, 42

35.



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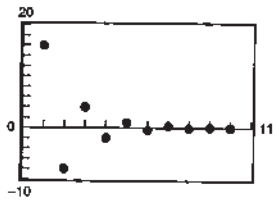

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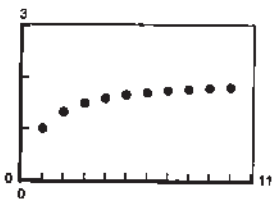


41. 9, 15, 21, 27, 33, 39, 45, 51, 57, 63

43. $6, 18, 36, 54, \frac{324}{5}, \frac{324}{5}, \frac{1944}{35}, \frac{1458}{35}, \frac{972}{35}, \frac{2916}{175}$

45. $3, \frac{5}{2}, \frac{7}{3}, \frac{9}{4}, \frac{11}{5}, \frac{13}{6}, \frac{15}{7}, \frac{17}{8}, \frac{19}{9}, \frac{21}{10}$ **47.** (c) **49.** (d)

51. $a_n = 3n - 2$ **53.** $a_n = n^2 - 1$ **55.** $a_n = \frac{n+1}{n+2}$

57. $a_n = \frac{(-1)^{n+1}}{2^n}$ **59.** $a_n = 1 + \frac{1}{n}$ **61.** $a_n = \frac{1}{n!}$

63. $(-1)^n + 2(1)^n = (-1)^n + 2$

65. 6, 8, 10, 12, 14; $a_n = 2n + 4$

67. 81, 27, 9, 3, 1; $a_n = \frac{243}{3^n}$ **69.** $\frac{1}{120}$ **71.** 90

73. 495 **75.** $n + 1$ **77.** $\frac{1}{2n(2n+1)}$ **79.** 35

81. 40 **83.** 30 **85.** $\frac{9}{5}$ **87.** 238 **89.** 30

91. 81 **93.** $\frac{47}{60}$ **95.** $\sum_{i=1}^{9} \frac{1}{3i} = 0.94299$

97. $\sum_{i=1}^{8} \left[2\left(\frac{i}{8}\right) + 3\right] = 33$ **99.** $\sum_{i=1}^{6} (-1)^{i+1} 3^i = -546$

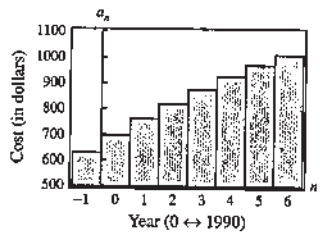
101. $\sum_{i=1}^{20} \frac{(-1)^{i+1}}{i^2} = 0.822$ **103.** $\sum_{i=1}^{5} \frac{2^i - 1}{2^{i+1}} = \frac{129}{64}$

105. $\frac{75}{16}$ **107.** $-\frac{3}{2}$ **109.** $\frac{2}{3}$ **111.** $\frac{1}{9}$

113. (a) $A_1 = \$5100.00$, $A_2 = \$5202.00$, $A_3 = \$5306.04$, $A_4 = \$5412.16$, $A_5 = \$5520.40$, $A_6 = \$5630.81$, $A_7 = \$5743.43$, $A_8 = \$5858.30$

(b) \$11,040.20

115.



According to the graph, hospital costs are increasing.

117. \$23,661.96 million; the sums are approximately the same.

119. True

121. 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144; $1, 2, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}, \frac{13}{8}, \frac{21}{13}, \frac{34}{21}, \frac{55}{34}, \frac{89}{55}$

123. a_n increases by $2(n - 1)$. The terms seem to be prime numbers, however, $a_{11} = 121$ is not prime.

125. $-\frac{x^3}{3}, \frac{x^5}{5}, -\frac{x^7}{7}, \frac{x^9}{9}, -\frac{x^{11}}{11}$

127. $-\frac{x^3}{6}, \frac{x^5}{120}, -\frac{x^7}{5040}, \frac{x^9}{362{,}880}, -\frac{x^{11}}{39{,}916{,}800}$

129. $\left[\begin{array}{ccc|c} 2 & 1 & 3 & -3 \\ -1 & 5 & 0 & 14 \\ -3 & -6 & -7 & -7 \end{array}\right]$

131. (a) $\begin{bmatrix} 10 & 19 \\ -12 & -5 \end{bmatrix}$ (b) $\begin{bmatrix} -30 & -45 \\ 28 & 4 \end{bmatrix}$

(c) $\begin{bmatrix} 56 & -43 \\ 48 & 114 \end{bmatrix}$ (d) $\begin{bmatrix} 48 & -72 \\ 36 & 122 \end{bmatrix}$

133. (a) $\begin{bmatrix} -1 & 0 & 0 \\ 2 & 0 & 4 \\ 1 & -1 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & -4 & 0 \\ -9 & -1 & -10 \\ -2 & 3 & -5 \end{bmatrix}$

(c) $\begin{bmatrix} 12 & 0 & -8 \\ 1 & 21 & 2 \\ -6 & -1 & 8 \end{bmatrix}$ (d) $\begin{bmatrix} 20 & 4 & 8 \\ 2 & 15 & -4 \\ 1 & -6 & 6 \end{bmatrix}$

135. -223 **137.** 664

Section 9.2 *(page 635)*

1. Arithmetic sequence, $d = -2$

3. Arithmetic sequence, $d = -\frac{1}{2}$

5. Arithmetic sequence, $d = 8$

7. Arithmetic sequence, $d = 0.6$

9. 21, 34, 47, 60, 73
Arithmetic sequence, $d = 13$

11. $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}$
Not an arithmetic sequence

13. 143, 136, 129, 122, 115
Arithmetic sequence, $d = -7$

15. $1, 4, \frac{7}{3}, \frac{7}{2}, \frac{13}{5}$
Not an arithmetic sequence

17. 15, 24, 33, 42, 51; $d = 9$; $a_n = 9n + 6$

19. $\frac{7}{2}, \frac{13}{4}, 3, \frac{11}{4}, \frac{5}{2}$; $d = -\frac{1}{4}$; $a_n = -\frac{1}{4}n + \frac{15}{4}$

21. 5, 11, 17, 23, 29 **23.** $-2.6, -3, -3.4, -3.8, -4.2$

25. $-2, 2, 6, 10, 14$ **27.** 22.45, 20.725, 19, 17.275, 15.55

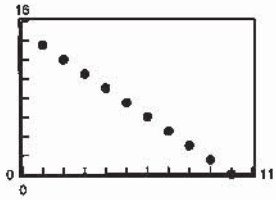
29. 59 **31.** -50 **33.** 18.6 **35.** $a_n = -2 + 3n$

37. $a_n = 108 - 8n$ **39.** $a_n = \frac{13}{2} - \frac{5}{2}n$

41. $a_n = \frac{10}{3}n + \frac{5}{3}$ **43.** $a_n = 103 - 3n$

45. (b) **47.** (c)

49.



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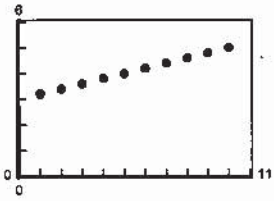


53. $-1, 3, 7, 11, 15, 19, 23, 27, 31, 35$

55. 19.25, 18.5, 17.75, 17, 16.25, 15.5, 14.75, 14, 13.25, 12.5

57. 1.505, 1.51, 1.515, 1.52, 1.525, 1.53, 1.535, 1.54, 1.545, 1.55

59. 890 **61.** 41 **63.** 4000 **65.** 1275

67. 25,250 **69.** 355 **71.** 126,750 **73.** 520

75. 44,625 **77.** 10,120 **79.** 10,000

81. (a) \$40,000 (b) \$217,500 **83.** 2340 seats

85. 405 bricks **87.** 585 seats **89.** 156 times

91. True **93.** $x, 3x, 5x, 7x, 9x, 11x, 13x, 15x, 17x, 19x$

95. (a) 4, 9, 16, 25, 36
(b) Sum of first n positive odd integers is n^2; 49
(c) $\frac{n}{2}[1 + (2n - 1)] = n^2$

97. $S_n + 5n$ **99.** $(2, -6, 3)$ **101.** 20 square units

103. $\frac{1}{3003}$

Section 9.3 *(page 644)*

1. Geometric sequence, $r = 3$

3. Not a geometric sequence

5. Geometric sequence, $r = -\frac{1}{2}$

7. Not a geometric sequence

9. Not a geometric sequence **11.** 8, 24, 72, 216, 648

13. $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$ **15.** $5, -\frac{1}{2}, \frac{1}{20}, -\frac{1}{200}, \frac{1}{2000}$

17. 3.5, 17.5, 87.5, 437.5, 2187.5 **19.** $1, e, e^2, e^3, e^4$

21. $64, 32, 16, 8, 4; \frac{1}{2}$
$a_n = 128\left(\frac{1}{2}\right)^n$

23. 4, 12, 36, 108, 324; 3
$a_n = \frac{4}{3}(3)^n$

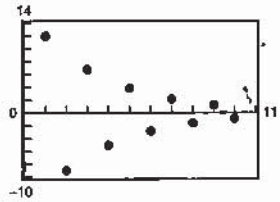
25. $6, -9, \frac{27}{2}, -\frac{81}{4}, \frac{243}{8}; -\frac{3}{2}$
$a_n = 6\left(-\frac{3}{2}\right)^{n-1}$

27. $\left(\frac{1}{2}\right)^7$ **29.** $-\frac{2}{3^{10}}$ **31.** $500(1.02)^{13}$ **33.** 9

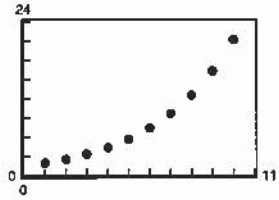
35. $-\frac{2}{9}$ **37.** 45,927 **39.** 50,388,480

41. 786,432 **43.** (a) **45.** (b)

47.



49.



51. 8, 4, 6, 5

53.

n	S_n
1	16
2	24
3	28
4	30
5	31
6	31.5
7	31.75
8	31.875
9	31.9375
10	31.96875

55. 511 **57.** 43

59. 29,921.31 **61.** 6.4 **63.** 2092.60

65. $\sum_{n=1}^{7} 5(3)^{n-1}$ **67.** $\sum_{n=1}^{7} 2\left(-\frac{1}{4}\right)^{n-1}$ **69.** 2 **71.** $\frac{2}{3}$

73. $\frac{16}{3}$ **75.** Series does not have a finite sum. **77.** $\frac{5}{3}$

79. -30 **81.** 32 **83.** $\frac{9}{4}$ **85.** $\frac{4}{11}$ **87.** $\frac{7}{22}$

89. (a) \$2158.92 (b) \$2191.12 (c) \$2208.04
(d) \$2219.64 (e) \$2225.35

91. \$26,050.85 **93.** \$7011.89 **95.** Answers will vary.

97. (a) \$26,198.27 (b) \$26,263.88

99. (a) \$637,678.02 (b) \$645,861.43

101. Answers will vary. **103.** 126 square inches

105. \$39.7 billion

107. (a) \$5,368,709.11 (b) \$10,737,418.23
(c) \$21,474,836.47

109. False. Any arithmetic sequence can be used as a counterexample.

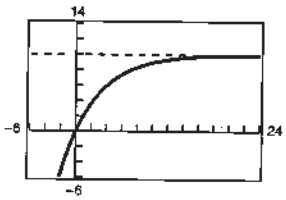
111. True. It is an arithmetic sequence with $d = 0$.

113. $8, \frac{16x}{3}, \frac{32x^2}{9}, \frac{64x^3}{27}, \frac{128x^4}{81}$

115. $\frac{1}{2}, \frac{7x}{2}, \frac{49x^2}{2}, \frac{343x^3}{2}, \frac{2401x^4}{2}$

117. $13{,}122e^{7x}$ **119.** $\frac{4096x^5}{243}$

121.



Horizontal asymptote: $y = 10$
Corresponds to the sum of the series

123. Divide the second term by the first to obtain the common ratio. The nth term is the first term times the common ratio raised to the $n - 1$ power.

125. 45.65 miles per hour **127.** 2.4 hours

129. $\begin{bmatrix} -33 & -3 \\ 27 & 1 \end{bmatrix}$ **131.** $\begin{bmatrix} 2 & 6 & -8 \\ 12 & 4 & -20 \\ 4 & 2 & 10 \end{bmatrix}$

133. 364 **135.** $\frac{227}{99}$

Section 9.4 *(page 654)*

1. $\frac{5}{(k+1)(k+2)}$ **3.** $\frac{(k+1)^2(k+4)^2}{6}$

5. $1 + 6 + 11 + \cdots + (5k - 4) + (5k + 1)$

7.–35. Answers will vary. **37.** 10, 40, 160, 640, 2560

39. 0, 2, 2, 6, 10

41. 2, 0, 3, 1, 4
First differences: $-2, 3, -2, 3$
Second differences: $5, -5, 5$
Neither

43. $-3, 6, -12, 24, -48$
First differences: $9, -18, 36, -72$
Second differences: $-27, 54, -108$
Neither

45. 2, 4, 16, 256, 65,536
First differences: 2, 12, 240, 65,280
Second differences: 10, 288, 65,040
Neither

47. 0, 4, 10, 18, 28
First differences: 4, 6, 8, 10
Second differences: 2, 2, 2
Quadratic

49. $0, -1, -2, -3, -4$
First differences: $-1, -1, -1, -1$
Second differences: 0, 0, 0
Linear

51. $a_n = n^2 - 2n + 7$ **53.** $a_n = \frac{7}{4}n^2 - 5n + 3$

55. False. The first differences are all the same.

57. Answers will vary. **59.** $(7, 5)$ **61.** $\left(-\frac{1}{2}, \frac{1}{4}\right), (2, 4)$

63. $(1, 2, 1)$ **65.** $(-1, 2, 4)$ **67.** 38

69. $4x^4 - 4x^2 + 1$ **71.** $-64x^3 + 240x^2 - 300x + 125$

Section 9.5 *(page 661)*

1. 21 **3.** 1 **5.** 15,504 **7.** 14 **9.** 4950

11. 4950 **13.** 35,960 **15.** 497,420 **17.** 749,398

19. 21 **21.** 56 **23.** $x^4 + 4x^3 + 6x^2 + 4x + 1$

25. $a^3 + 9a^2 + 27a + 27$

27. $y^4 - 8y^3 + 24y^2 - 32y + 16$

29. $x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$

31. $r^6 + 12r^5s + 60r^4s^2 + 160r^3s^3 + 240r^2s^4 + 192rs^5 + 64s^6$

33. $x^5 - 5x^4y + 10x^3y^2 - 10x^2y^3 + 5xy^4 - y^5$

35. $1 - 12x + 48x^2 - 64x^3$

37. $x^8 + 20x^6 + 150x^4 + 500x^2 + 625$

39. $\frac{1}{x^5} + \frac{5y}{x^4} + \frac{10y^2}{x^3} + \frac{10y^3}{x^2} + \frac{5y^4}{x} + y^5$

41. $2x^4 - 24x^3 + 113x^2 - 246x + 207$

43. $-4x^6 - 24x^5 - 60x^4 - 83x^3 - 42x^2 - 60x + 20$

45. $243t^5 - 405t^4s + 270t^3s^2 - 90t^2s^3 + 15ts^4 - s^5$

47. $81 - 216z + 216z^2 - 96z^3 + 16z^4$ **49.** 3,247,695

51. 180 **53.** $-489{,}888$ **55.** 210

57. $x^2 + 20x^{3/2} + 150x + 500x^{1/2} + 625$

59. $x^2 - 3x^{4/3}y^{1/3} + 3x^{2/3}y^{2/3} - y$

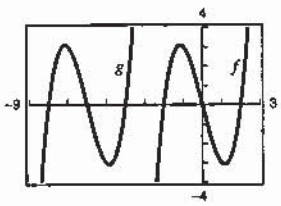
61. $3x^2 + 3xh + h^2,\ h \neq 0$

63. $\dfrac{\sqrt{x+h} - \sqrt{x}}{h} = \dfrac{1}{\sqrt{x+h} + \sqrt{x}},\ h \neq 0$

65. -4 **67.** $2035 + 828i$ **69.** 1 **71.** 1.172

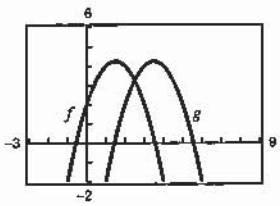
73. 510,568.785

75.



g is shifted 6 units left of f.
$g(x) = x^3 + 18x^2 + 104x + 192$

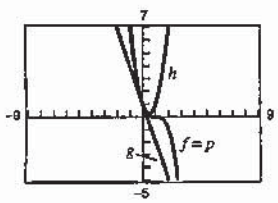
77.



g is shifted 2 units right of f.
$g(x) = -x^2 + 7x - 8$

79. (a) 792 (b) 36 (c) 792 (d) 12

81.

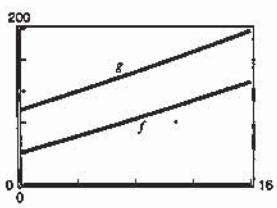


$p(x)$ is the expansion of $f(x)$.

83. 0.273 **85.** 0.171

87. (a) $g(t) = 0.0348t^2 + 5.8043t + 95.588$

(b)



89. False. The correct term is $126{,}720x^4y^8$.

91. The first and last numbers in each row are 1. Every other number in each row is formed by adding the two numbers immediately above the number.

93. $n + 1$ terms **95. and 97.** Answers will vary.

99. $g(x)$ is shifted 8 units up from $f(x)$.

101. $g(x)$ is the reflection of $f(x)$ in the y-axis.

103. $\begin{bmatrix} -2 & -1 & -3 \\ 8 & 0 & 4 \\ -1 & -2 & 7 \end{bmatrix}$ **105.** $\begin{bmatrix} 6 & 11 & 15 \\ -30 & -2 & -16 \\ 13 & 10 & -33 \end{bmatrix}$

107. $\begin{bmatrix} 9 & 11 & 12 \\ -13 & -25 & -5 \\ -5 & -18 & -6 \end{bmatrix}$ **109.** $\begin{bmatrix} 4 & -5 \\ 5 & -6 \end{bmatrix}$

Section 9.6 *(page 671)*

1. 6 **3.** 5 **5.** 3 **7.** 9 **9.** 192 **11.** 18

13. 3,628,800 **15.** 12 **17.** 6,760,000

19. (a) 900 (b) 648 (c) 180 (d) 600

21. 64,000 **23.** (a) 720 (b) 48 **25.** 24

27. 336 **29.** 120 **31.** $n = 5$ or $n = 6$

33. 27,907,200 **35.** 197,149,680 **37.** 4845

39. 120 **41.** (a) 16 (b) 14 **43.** 420 **45.** 2520

47. ABCD, ABDC, ACBD, ACDB, ADBC, ADCB, BACD, BADC, CABD, CADB, DABC, DACB, BCAD, BDAC, CBAD, CDAB, DBAC, DCAB, BCDA, BDCA, CBDA, CDBA, DBCA, DCBA

49. AB, AC, AD, AE, AF, BC, BD, BE, BF, CD, CE, CF, DE, DF, EF

51. 4845 **53.** 3,838,380 **55.** 75,287,520 **57.** 36

59. (a) 495 (b) 210 **61.** (a) 70 (b) 54 (c) 16

63. 5 **65.** 20

67. False. Order matters in a permutation.

69. False. ${}_nP_r = {}_nC_r$ if $r = 1$ or 0.

71. ${}_nP_r$ represents the number of ways to choose and order r elements out of a collection of n elements.

73. (b). Numerous permutations can be made from each combination.

75. and 77. Answers will vary. **79.** $\frac{11}{2}$ **81.** 8.32

83. $(6, -13)$ **85.** $(-3, 4)$

87. $x^6 - 6x^5 + 15x^4 - 20x^3 + 15x^2 - 6x + 1$

89. $81x^4 - 108x^3y + 54x^2y^2 - 12xy^3 + y^4$

Section 9.7 *(page 682)*

1. $\{(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6), (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)\}$

3. {ABC, ACB, BAC, BCA, CAB, CBA}

5. {(A, B), (A, C), (A, D), (A, E), (B, C), (B, D), (B, E), (C, D), (C, E), (D, E)}

7. $\frac{3}{8}$ **9.** $\frac{7}{8}$ **11.** $\frac{3}{13}$ **13.** $\frac{3}{26}$ **15.** $\frac{1}{9}$ **17.** $\frac{35}{36}$

19. $\frac{1}{6}$ **21.** $\frac{1}{5}$ **23.** $\frac{2}{5}$ **25.** 0.3 **27.** $\frac{2}{3}$

29. 0.85 **31.** $\frac{7}{20}$

33. (a) 78,000 (b) 0.3 (c) 0.37 (d) 0.1

35. (a) 0.58 (b) 0.956 (c) 0.004

37. (a) $\frac{672}{1254}$ (b) $\frac{582}{1254}$ (c) $\frac{548}{1254}$

39. $P(\{\text{Taylor wins}\}) = \frac{1}{2}$

$P(\{\text{Moore wins}\}) = P(\{\text{Perez wins}\}) = \frac{1}{4}$

41. (a) $\frac{21}{1292} \approx 0.016$ (b) $\frac{225}{646} \approx 0.348$ (c) $\frac{49}{323} \approx 0.152$

43. (a) $\frac{1}{3}$ (b) $\frac{5}{8}$ **45.** (a) $\frac{1}{120}$ (b) $\frac{1}{24}$

47. (a) 0.346 (b) 0.0000029

49. (a) $\frac{14}{55}$ (b) $\frac{12}{55}$ (c) $\frac{54}{55}$

51. (a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{9}{100}$ (d) $\frac{1}{30}$

53. (a) 0.9702 (b) 0.9998 (c) 0.0002

55. (a) $\dfrac{1}{15{,}625}$ (b) $\dfrac{4096}{15{,}625}$ (c) $\dfrac{11{,}529}{15{,}625}$

57. 0.1024 **59.** $\frac{7}{16}$ **61.** True

63. (a) As you consider successive people with distinct birthdays, the probabilities must decrease to take into account the birth dates already used. Because the birth dates of people are independent events, multiply the respective probabilities of distinct birthdays.

(b) $\frac{365}{365} \cdot \frac{364}{365} \cdot \frac{363}{365} \cdot \frac{362}{365}$

(c) Answers will vary.

(d) Q_n is the probability that the birthdays are *not* distinct, which is equivalent to at least two people having the same birthday.

(e)

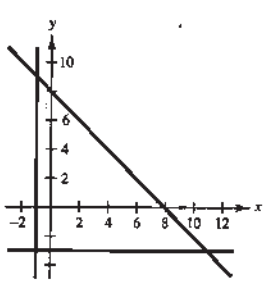
n	10	15	20	23	30	40	50
P_n	0.88	0.75	0.59	0.49	0.29	0.11	0.03
Q_n	0.12	0.25	0.41	0.51	0.71	0.89	0.97

(f) 23

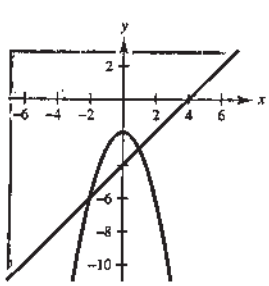
65. $x = \frac{22}{4} = \frac{11}{2}$ **67.** $x = -10$ **69.** $x = e^8$

71. $x = \frac{1}{6}e^4$

73.



75.

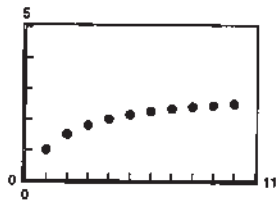


77. 15 **79.** 165

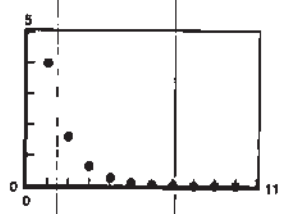
Review Exercises *(page 688)*

1. 8, 5, 4, $\frac{7}{2}$, $\frac{16}{5}$ **3.** 5, $\frac{10}{3}$, 3, $\frac{20}{7}$, $\frac{25}{9}$ **5.** 72, 36, 12, 3, $\frac{3}{5}$

7.



9.



11. $\frac{1}{380}$ **13.** 1 **15.** 30 **17.** $\frac{205}{24}$ **19.** 6050

21. 418 **23.** $\displaystyle\sum_{k=1}^{20} \frac{1}{2k}$ **25.** $\displaystyle\sum_{k=1}^{9} \frac{k}{k+1}$

27. (a) $\frac{1111}{2000}$ (b) $\frac{5}{9}$ **29.** (a) $\frac{2{,}020{,}202}{100{,}000{,}000}$ (b) $\frac{2}{99}$

31. (a) $a_1 = 2550$, $a_2 = 2601$, $a_3 = 2653$, $a_4 = 2706.10$, $a_5 = 2760.20$, $a_6 = 2815.40$, $a_7 = 2871.70$, $a_8 = 2929.10$

(b) \$5520.10

33. Arithmetic sequence, $d = -7$

35. Arithmetic sequence, $d = \frac{1}{2}$ **37.** 3, 7, 11, 15, 19

39. 1, 4, 7, 10, 13

41. 35, 32, 29, 26, 23; -3 **43.** 9, 16, 23, 30, 37; 7

$a_n = 38 - 3n$ $a_n = 2 + 7n$

45. $a_n = 103 - 3n$; 1430 **47.** 80 **49.** 88

51. 25,250 **53.** (a) \$43,000 (b) \$192,500

55. 4, -1, $\frac{1}{4}$, $-\frac{1}{16}$, $\frac{1}{64}$ **57.** 9, 6, 4, $\frac{8}{3}$, $\frac{16}{9}$ or 9, -6, 4, $-\frac{8}{3}$, $\frac{16}{9}$

59. 120, 40, $\frac{40}{3}$, $\frac{40}{9}$, $\frac{40}{27}$; $\frac{1}{3}$ **61.** 25, -15, 9, $-\frac{27}{5}$, $\frac{81}{25}$; $-\frac{3}{5}$

$a_n = 120\left(\frac{1}{3}\right)^{n-1}$ $a_n = 25\left(-\frac{3}{5}\right)^{n-1}$

63. $a_n = 16\left(-\frac{1}{2}\right)^{n-1}$; 10.67

65. $a_n = 100(1.05)^{n-1}$; 3306.60 **67.** 127 **69.** 3277

71. 1301.01 **73.** 24.85 **75.** 5486.45 **77.** 8

79. 12 **81.** (a) $a_t = 120{,}000(0.7)^t$ (b) \$20,168.40

83. \$3909.96 **85. and 87.** Answers will vary.

89. 465 **91.** 4676

93. 5, 10, 15, 20, 25

First differences: 5, 5, 5, 5

Second differences: 0, 0, 0

Linear model

95. 16, 15, 14, 13, 12

First differences: -1, -1, -1, -1

Second differences: 0, 0, 0

Linear model

97. 45 **99.** 126 **101.** 20 **103.** 70

105. $a^5 - 15a^4b + 90a^3b^2 - 270a^2b^3 + 405ab^4 - 243b^5$

107. $\frac{x^4}{16} + \frac{x^3y}{2} + \frac{3x^2y^2}{2} + 2xy^3 + y^4$ **109.** $41 + 840i$

111. 10 **113.** 48 **115.** 5040 **117.** 3,628,800

119. 15,504 **121.** $\frac{1}{9}$

123. $P(\{3\}) = \frac{1}{6}$

$P(\{(1, 5), (5, 1), (2, 4), (4, 2), (3, 3)\}) = \frac{5}{36}$

There is a higher probability of rolling a 3 with one die.

125. (a) 0.416 (b) 0.8 (c) 0.074 **127.** 0.512

129. $\frac{31}{32}$ **131.** True

133. (a) Odd-numbered terms are negative.

(b) Even-numbered terms are negative.

135. (a) Arithmetic. There is a constant difference between consecutive terms.

(b) Geometric. Each term is a constant multiple of the preceding term. In this case the common ratio is greater than 1.

137. Each term of the sequence is defined using a previous term or terms.

139. (d) **141.** (b)

143.

n	100	500	1000	5000
a_n	2.704	2.7156	2.7169	2.7180

n	10,000	15,000	20,000	25,000
a_n	2.71814	2.71819	2.71821	2.71823

a_n approaches e.

145. $0 \le P \le 1$; closed

147. Meteorological records gathered over an extended period of time indicate that under similar weather conditions it will rain 60% of the time.

Chapter Test *(page 694)*

1. $1, -\frac{2}{3}, \frac{4}{9}, -\frac{8}{27}, \frac{16}{81}$ **2.** 12, 16, 20, 24, 28 **3.** 7920

4. $a_n = 5100 - 100n$ **5.** $a_n = 4\left(\frac{1}{2}\right)^{n-1}$

6. $\sum_{n=1}^{12} \frac{2}{3n+1}$ **7.** 3825 **8.** 189 **9.** 28.80

10. $\frac{50}{9}$ **11.** \$47,868.33 **12.** Answers will vary.

13. 84 **14.** 1140 **15.** 8568 **16.** 780

17. 56 **18.** 26,000 **19.** 12,650 **20.** $\frac{3}{26}$ **21.** $\frac{1}{6}$

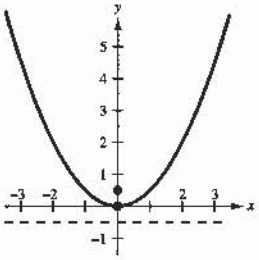
22. (a) $\frac{1}{4}$ (b) $\frac{121}{3600}$ (c) $\frac{1}{60}$

Chapter 10

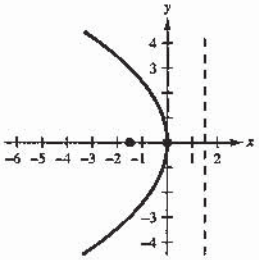
Section 10.1 *(page 701)*

1. (e) **3.** (d) **5.** (a)

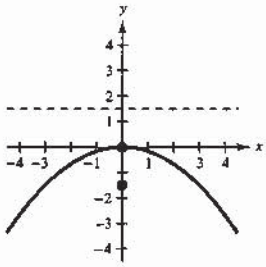
7. Vertex: (0, 0)
Focus: $\left(0, \frac{1}{2}\right)$
Directrix: $y = -\frac{1}{2}$



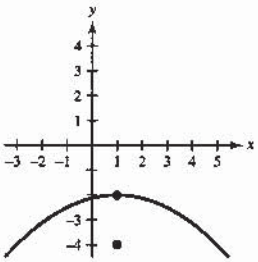
9. Vertex: (0, 0)
Focus: $\left(-\frac{3}{2}, 0\right)$
Directrix: $x = \frac{3}{2}$



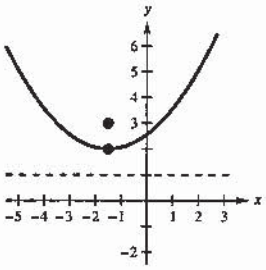
11. Vertex: (0, 0)
Focus: $\left(0, -\frac{3}{2}\right)$
Directrix: $y = \frac{3}{2}$



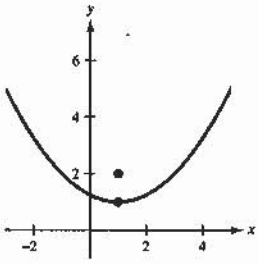
13. Vertex: (1, −2)
Focus: (1, −4)
Directrix: $y = 0$



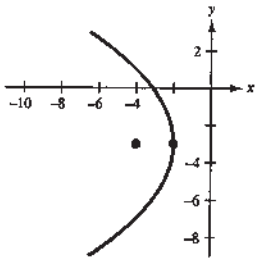
15. Vertex: $\left(-\frac{3}{2}, 2\right)$
Focus: $\left(-\frac{3}{2}, 3\right)$
Directrix: $y = 1$



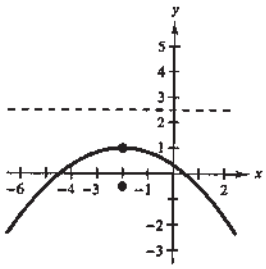
17. Vertex: (1, 1)
Focus: (1, 2)
Directrix: $y = 0$



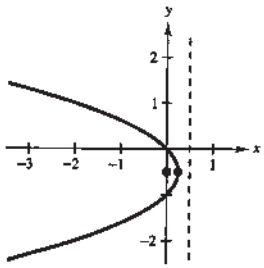
19. Vertex: $(-2, -3)$
Focus: $(-4, -3)$
Directrix: $x = 0$



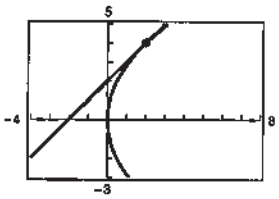
21. Vertex: $(-2, 1)$
Focus: $\left(-2, -\frac{1}{2}\right)$
Directrix: $y = \frac{5}{2}$



23. Vertex: $\left(\frac{1}{4}, -\frac{1}{2}\right)$
Focus: $\left(0, -\frac{1}{2}\right)$
Directrix: $x = \frac{1}{2}$

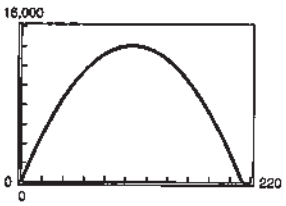


25. $y = -\sqrt{-6x}$ **27.** $y = \frac{2}{3}x^2$ **29.** $x^2 = -6y$
31. $y^2 = -8x$ **33.** $x^2 = 4y$ **35.** $y^2 = -8x$
37. $y^2 = 9x$ **39.** $(x - 3)^2 = -(y - 1)$
41. $y^2 = 2(x + 2)$ **43.** $(y - 2)^2 = -8(x - 5)$
45. $x^2 = 8(y - 4)$ **47.** $(y - 2)^2 = 8x$
49.



$(2, 4)$

51. $4x - y - 8 = 0$; $(2, 0)$ **53.** $4x - y + 2 = 0$; $\left(-\frac{1}{2}, 0\right)$
55.

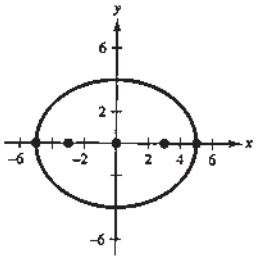


About 106 sales

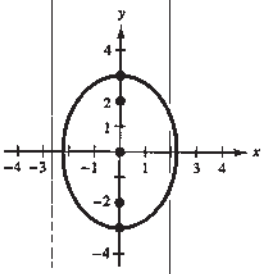
57. $y = \frac{1}{14}x^2$ **59.** (a) $x^2 = -640y$ (b) 8 feet
61. (a) $17{,}500\sqrt{2}$ miles per hour
(b) $x^2 = -16{,}400(y - 4100)$
63. (a) $y = -\frac{1}{64}x^2 + 75$ (b) 69.3 feet
65. False. If the graph intersected the directrix, there would exist points nearer the directrix then the focus.
67. $\pm 1, \pm 2, \pm 4$ **69.** $\pm\frac{1}{2}, \pm 1, \pm 2, \pm 4, \pm 8, \pm 16$

Section 10.2 *(page 710)*

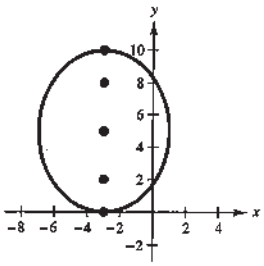
1. (b) **3.** (d) **5.** (a)
7. Center: $(0, 0)$
Vertices: $(\pm 5, 0)$
Foci: $(\pm 3, 0)$
Eccentricity: $\frac{3}{5}$



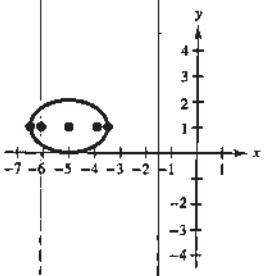
9. Center: $(0, 0)$
Vertices: $(0, \pm 3)$
Foci: $(0, \pm 2)$
Eccentricity: $\frac{2}{3}$



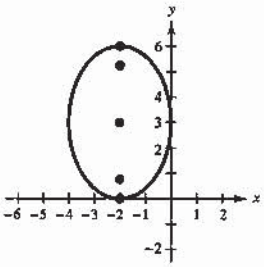
11. Center: $(-3, 5)$
Vertices: $(-3, 0), (-3, 10)$
Foci: $(-3, 8), (-3, 2)$
Eccentricity: $\frac{3}{5}$



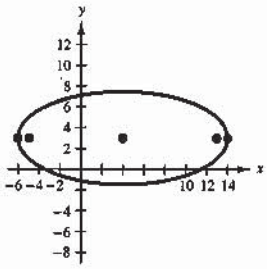
13. Center: $(-5, 1)$
Vertices:
$$\left(-\frac{7}{2}, 1\right), \left(-\frac{13}{2}, 1\right)$$
Foci: $\left(-5 \pm \frac{\sqrt{5}}{2}, 1\right)$
Eccentricity: $\frac{\sqrt{5}}{3}$



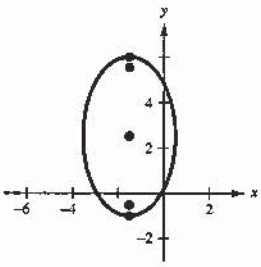
15. Center: $(-2, 3)$
Vertices:
$(-2, 6), (-2, 0)$
Foci: $(-2, 3 \pm \sqrt{5})$
Eccentricity: $\frac{\sqrt{5}}{3}$



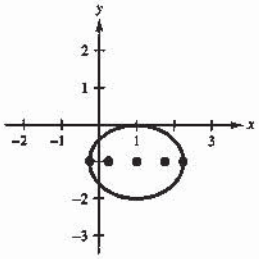
17. Center: $(4, 3)$
Vertices:
$(-6, 3), (14, 3)$
Foci: $(4 \pm 4\sqrt{5}, 3)$
Eccentricity: $\frac{2\sqrt{5}}{5}$



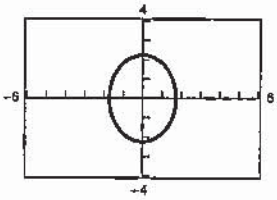
19. Center: $\left(-\frac{3}{2}, \frac{5}{2}\right)$
Vertices: $\left(-\frac{3}{2}, \frac{5 \pm 4\sqrt{3}}{2}\right)$
Foci: $\left(-\frac{3}{2}, \frac{5}{2} \pm 2\sqrt{2}\right)$
Eccentricity: $\frac{\sqrt{6}}{3}$



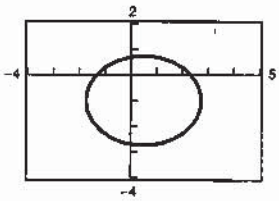
21. Center: $(1, -1)$
Vertices: $\left(\frac{9}{4}, -1\right), \left(-\frac{1}{4}, -1\right)$
Foci: $\left(\frac{7}{4}, -1\right), \left(\frac{1}{4}, -1\right)$
Eccentricity: $\frac{3}{5}$



23. Center: $(0, 0)$
Vertices: $(0, \pm\sqrt{5})$
Foci: $(0, \pm\sqrt{2})$



25. Center: $\left(\frac{1}{2}, -1\right)$
Vertices: $\left(\frac{1}{2} \pm \sqrt{5}, -1\right)$
Foci: $\left(\frac{1}{2} \pm \sqrt{2}, -1\right)$



27. $\frac{x^2}{4} + \frac{y^2}{16} = 1$ **29.** $\frac{x^2}{36} + \frac{y^2}{32} = 1$

31. $\frac{x^2}{36} + \frac{y^2}{11} = 1$ **33.** $\frac{21x^2}{400} + \frac{y^2}{25} = 1$

35. $\frac{(x-2)^2}{1} + \frac{(y-3)^2}{9} = 1$

37. $\frac{(x+2)^2}{16} + \frac{(y-3)^2}{9} = 1$

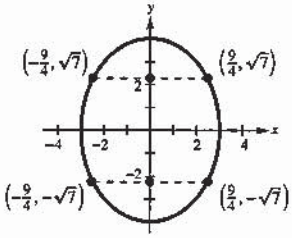
39. $\frac{(x-2)^2}{4} + \frac{(y-4)^2}{1} = 1$ **41.** $\frac{x^2}{48} + \frac{(y-4)^2}{64} = 1$

43. $\frac{(x-3)^2}{9} + \frac{(y-5)^2}{16} = 1$ **45.** $\frac{x^2}{16} + \frac{(y-4)^2}{12} = 1$

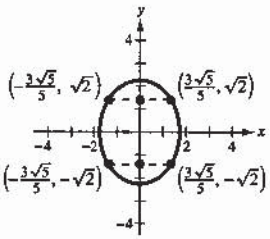
47. $\frac{x^2}{25} + \frac{y^2}{16} = 1$ **49.** $(\pm\sqrt{5}, 0)$; 6 feet **51.** 40

53. $\frac{x^2}{4.88} + \frac{y^2}{1.39} = 1$ **55.** Answers will vary.

57.



59.



61. True **63.** False

65. (a) $A = ma(20 - a)$

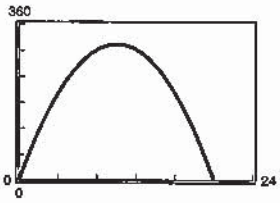
(b) $\frac{x^2}{196} + \frac{y^2}{36} = 1$

(c)

a	8	9	10	11	12	13
A	301.6	311.0	314.2	311.0	301.6	285.9

$a = 10$. Circle

(d)



$a = 10$

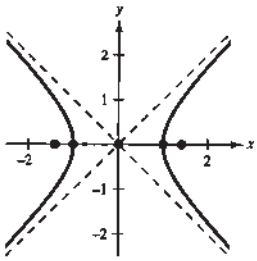
67. Geometric **69.** Arithmetic **71.** $a_n = -\frac{1}{4}n + \frac{1}{4}$

73. $a_n = 9n$ **75.** 547 **77.** 340.1545

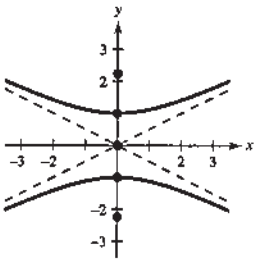
Section 10.3 *(page 720)*

1. (b) **3.** (a)

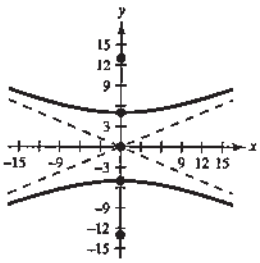
5. Center: $(0, 0)$
Vertices: $(\pm 1, 0)$
Foci: $(\pm\sqrt{2}, 0)$
Asymptotes: $y = \pm x$



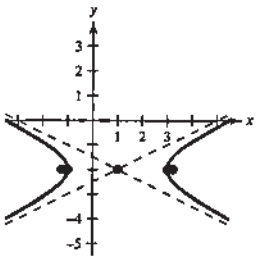
7. Center: $(0, 0)$
Vertices: $(0, \pm 1)$
Foci: $(0, \pm\sqrt{5})$
Asymptotes: $y = \pm\frac{1}{2}x$



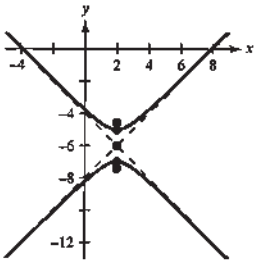
9. Center: $(0, 0)$
Vertices: $(0, \pm 5)$
Foci: $(0, \pm\sqrt{106})$
Asymptotes: $y = \pm\frac{5}{9}x$



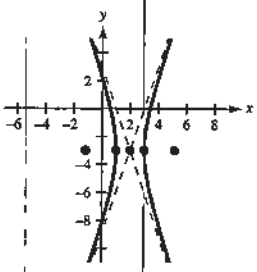
11. Center: $(1, -2)$
Vertices: $(3, -2), (-1, -2)$
Foci: $(1 \pm \sqrt{5}, -2)$
Asymptotes:
$y = -2 \pm \frac{1}{2}(x - 1)$



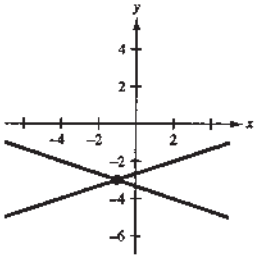
13. Center: $(2, -6)$
Vertices: $(2, -5), (2, -7)$
Foci: $(2, -6 \pm \sqrt{2})$
Asymptotes:
$y = -6 \pm (x - 2)$



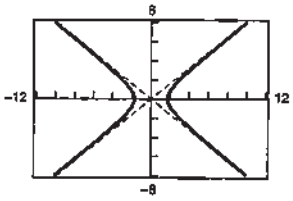
15. Center: $(2, -3)$
Vertices: $(3, -3), (1, -3)$
Foci: $(2 \pm \sqrt{10}, -3)$
Asymptotes:
$y = -3 \pm 3(x - 2)$



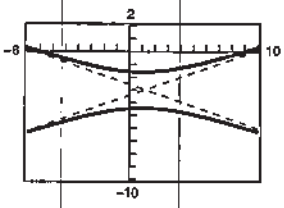
17. The graph of this equation is two lines intersecting at $(-1, -3)$.



19. Center: $(0, 0)$
Vertices: $(\pm\sqrt{3}, 0)$
Foci: $(\pm\sqrt{5}, 0)$



21. Center: $(1, -3)$
Vertices: $(1, -3 \pm \sqrt{2})$
Foci: $(1, -3 \pm 2\sqrt{5})$



23. $\dfrac{y^2}{4} - \dfrac{x^2}{12} = 1$ **25.** $\dfrac{x^2}{1} - \dfrac{y^2}{25} = 1$

27. $\dfrac{17y^2}{1024} - \dfrac{17x^2}{64} = 1$ **29.** $\dfrac{(x-4)^2}{4} - \dfrac{y^2}{12} = 1$

31. $\dfrac{(y-5)^2}{16} - \dfrac{(x-4)^2}{9} = 1$ **33.** $\dfrac{y^2}{9} - \dfrac{4(x-2)^2}{9} = 1$

35. $\dfrac{(y-2)^2}{4} - \dfrac{x^2}{4} = 1$ **37.** $\dfrac{(x-2)^2}{1} - \dfrac{(y-2)^2}{1} = 1$

39. $\dfrac{(x-3)^2}{9} - \dfrac{(y-2)^2}{4} = 1$ **41.** $(3300, -2750)$

43. $(12(\sqrt{5} - 1), 0) \approx (14.83, 0)$ **45.** Ellipse

47. Parabola **49.** Hyperbola **51.** Circle

53. False. $b = 0$ yields an undefined term. For the trivial solution of two intersecting lines to occur, the standard form of the equation of the hyperbola would be equal to zero.

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 0 \text{ or } \frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 0.$$

55. Answers will vary. **57.** $3x^2 + \frac{23}{2}x - 2$

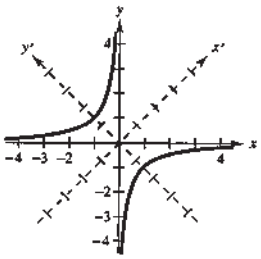
59. $x^2 + 2xy + y^2 + 6x + 6y + 9$ **61.** $(x + 7)^2$

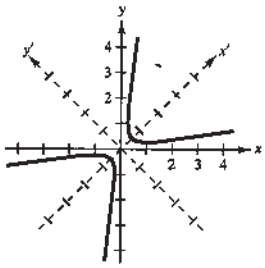
63. $x(3x + 2)(2x - 5)$ **65.** $(4 - x)(x^2 + 1)$

Section 10.4 *(page 729)*

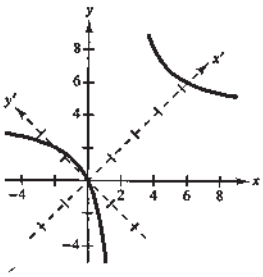
1. $(-4, 0)$ **3.** $\left(\frac{1}{2}(\sqrt{3} - 6), \frac{1}{2}(1 + 6\sqrt{3})\right)$

5. $\dfrac{(y')^2}{2} - \dfrac{(x')^2}{2} = 1$ **7.** $\dfrac{(x')^2}{1/3} - \dfrac{(y')^2}{1/5} = 1$

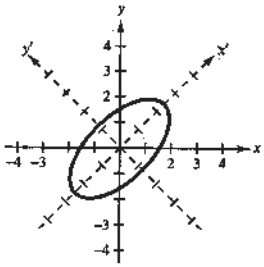


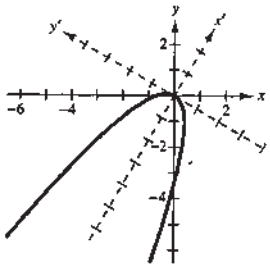


9. $\dfrac{(x' - 3\sqrt{2})^2}{16} - \dfrac{(y' - \sqrt{2})^2}{16} = 1$

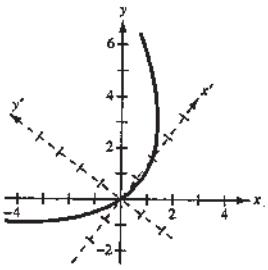


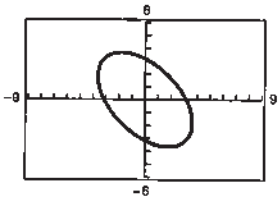
11. $\dfrac{(x')^2}{6} + \dfrac{(y')^2}{3/2} = 1$ **13.** $x' = -(y')^2$





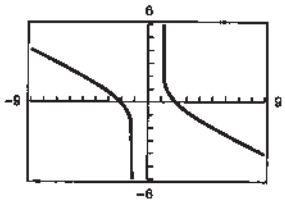
15. $y' = \frac{1}{6}(x')^2 - \frac{1}{3}x'$ **17.**





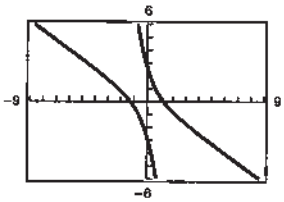
$\theta = 45°$

19.



$\theta = 26.57°$

21.



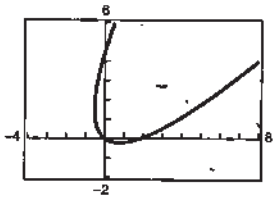
$\theta = 31.72°$

23. (e) **25.** (b) **27.** (d)

29. (a) Parabola

(b) $y = \dfrac{24x + 40 \pm \sqrt{3000x + 1600}}{18}$

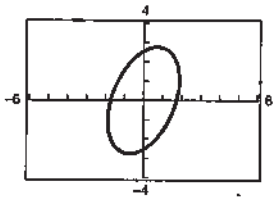
(c)



31. (a) Ellipse

(b) $y = \dfrac{8x \pm \sqrt{-356x^2 + 1260}}{14}$

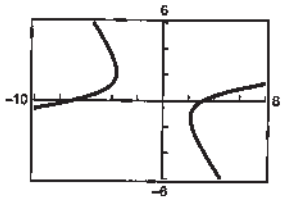
(c)



33. (a) Hyperbola

(b) $y = \dfrac{6x \pm \sqrt{56x^2 + 80x - 440}}{-10}$

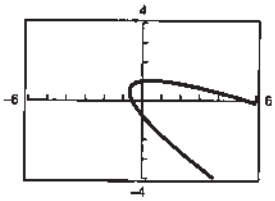
(c)



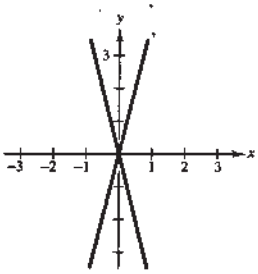
35. (a) Parabola

(b) $y = \dfrac{-4x + 1 \pm \sqrt{72x + 49}}{8}$

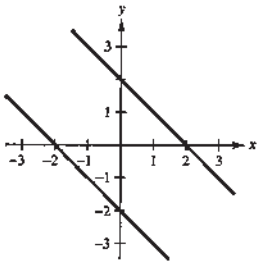
(c)



37.



39.



41. $(2, 2), (2, 4)$ **43.** $(-8, 12)$ **45.** $(0, 8), (12, 8)$

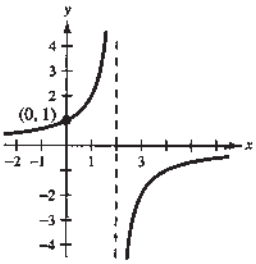
47. $(0, 4)$ **49.** $(1, \sqrt{3}), (1, -\sqrt{3})$ **51.** No solution

53. $(-3, 0), \left(0, \frac{3}{2}\right)$

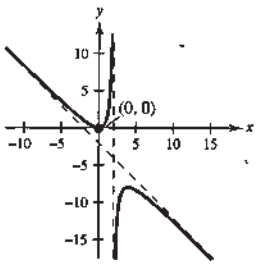
55. True. The discriminant will be greater than zero.

57. Answers will vary.

59.



61.



63. (a) $\begin{bmatrix} -15 & 9 \\ 25 & 7 \end{bmatrix}$ (b) $\begin{bmatrix} 12 & 30 \\ 3 & -20 \end{bmatrix}$ (c) $\begin{bmatrix} -5 & -18 \\ 12 & 19 \end{bmatrix}$

65. (a) $[45]$ (b) $\begin{bmatrix} 12 & -6 & 15 \\ -16 & 8 & -20 \\ 20 & -10 & 25 \end{bmatrix}$ (c) Not possible

67. 688,128 **69.** 53,760

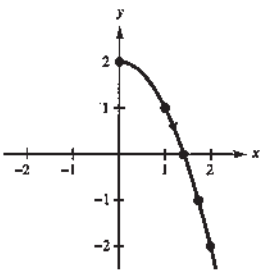
Section 10.5 *(page 736)*

1. (c) **3.** (b) **5.** (a) **7.** (f)

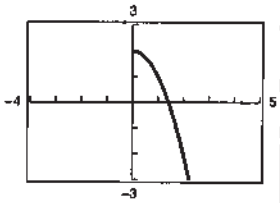
9. (a)

t	0	1	2	3	4
x	0	1	1.414	1.732	2
y	2	1	0	-1	-2

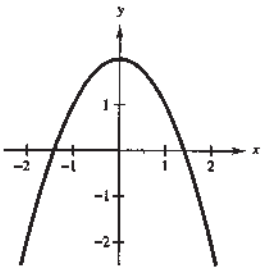
(b)



(c)

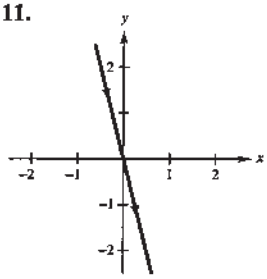


(d) $y = 2 - x^2$



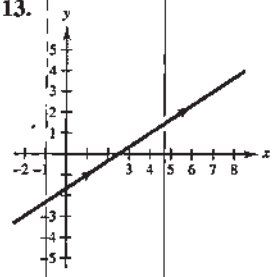
The graph is an entire parabola rather than just the right half.

11.



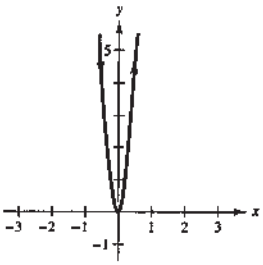
$y = -4x$

13.



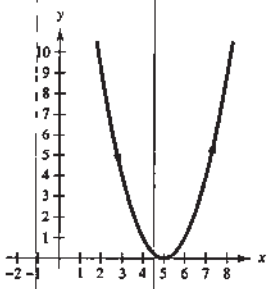
$y = \frac{2}{3}x - \frac{5}{3}$

15.



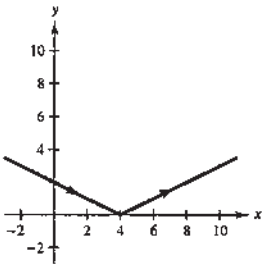
$y = 16x^2$

17.



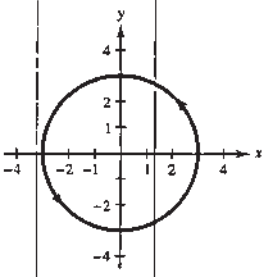
$y = (x - 5)^2$

19.



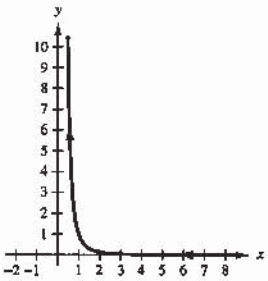
$y = \frac{1}{2}|x - 4|$

21.



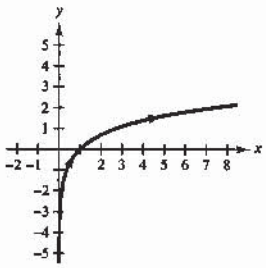
$x^2 + y^2 = 9$

23.



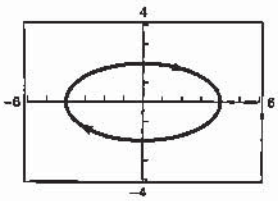
$y = e^{-3 \ln x} = x^{-3},\ x > 0$

25.

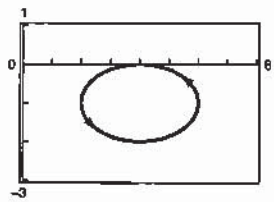


$y = \ln x$

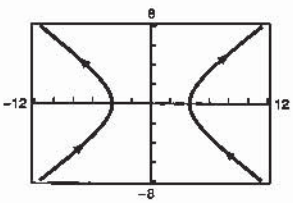
27.



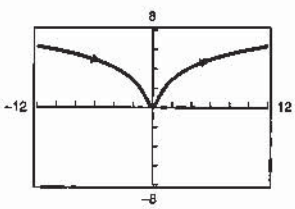
29.



31.



33.



35. Each curve represents a portion of the line $y = 2x + 1$.

	Domain	*Orientation*
(a)	$(-\infty, \infty)$	Left to right
(b)	$[-1, 1]$	Depends on θ
(c)	$(0, \infty)$	Right to left
(d)	$(0, \infty)$	Left to right

37. $y - y_1 = \dfrac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

39. $\dfrac{(x-h)^2}{a^2} + \dfrac{(y-k)^2}{b^2} = 1$

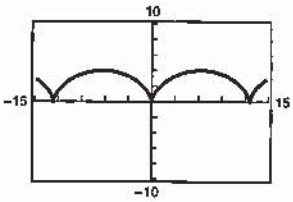
41. $x = 5t$
$y = -2t$

43. $x = 2 + 4\cos\theta$
$y = 1 + 4\sin\theta$

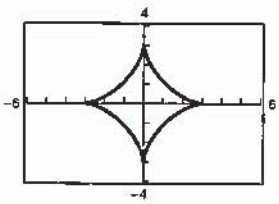
45. $x = 5\cos\theta$
$y = 3\sin\theta$

47. $x = t,\ y = 3t - 2$
$x = t^3,\ y = 3t^3 - 2$

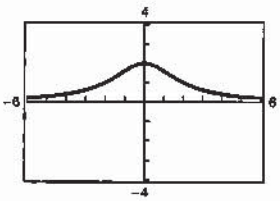
49.



51.



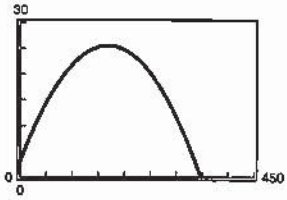
53.



55. (b) **57.** (d)

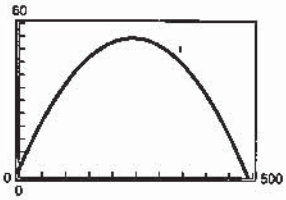
59. (a) $x = (146.67\cos\theta)t$
$y = 3 + (146.67\sin\theta)t - 16t^2$

(b) $x = 141.7t$
$y = 3 + 38.0t - 16t^2$



No

(c) $x = 135.0t$
$y = 3 + 57.3t - 16t^2$



Yes

(d) About 19.4°

61. True **63.** Only the direction of the curve would change.

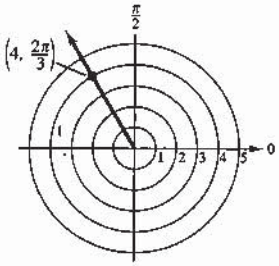
65. $x = 3 \pm \sqrt{5}$ **67.** $x = \pm\sqrt{9 + \sqrt{63}}, \pm\sqrt{9 - \sqrt{63}}$

69. 18,500 **71.** $-\dfrac{11{,}935}{12}$ **73.** ≈ 29.6532

Section 10.6 *(page 743)*

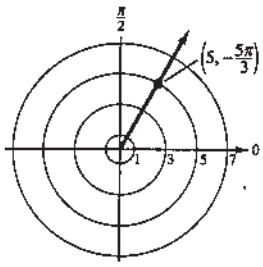
1. $(0, 4)$ **3.** $\left(\dfrac{\sqrt{2}}{2}, \dfrac{\sqrt{2}}{2}\right)$

5.



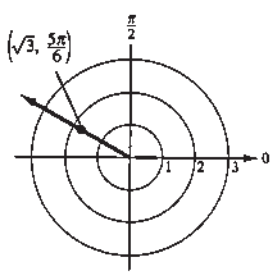
$\left(-4, -\dfrac{\pi}{3}\right), \left(-4, \dfrac{5\pi}{3}\right), \left(4, -\dfrac{4\pi}{3}\right)$

7.



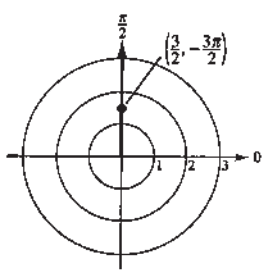
$\left(5, \frac{\pi}{3}\right), \left(-5, \frac{4\pi}{3}\right), \left(-5, -\frac{2\pi}{3}\right)$

9.



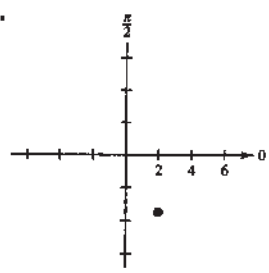
$\left(-\sqrt{3}, \frac{11\pi}{6}\right), \left(\sqrt{3}, -\frac{7\pi}{6}\right), \left(-\sqrt{3}, -\frac{\pi}{6}\right)$

11.



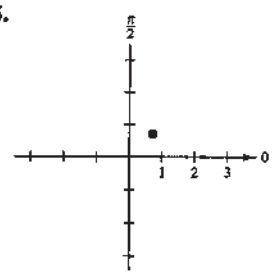
$\left(\frac{3}{2}, \frac{\pi}{2}\right), \left(-\frac{3}{2}, \frac{3\pi}{2}\right), \left(-\frac{3}{2}, -\frac{\pi}{2}\right)$

13.



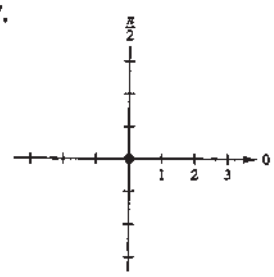
$(2, -2\sqrt{3})$

15.



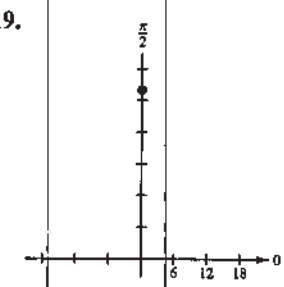
$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

17.



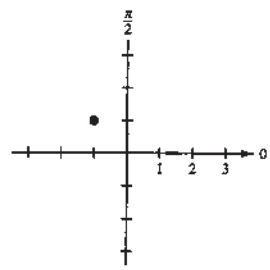
$(0, 0)$

19.



$(0, 32)$

21.

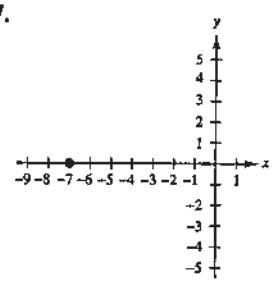


$(-1.004, 0.996)$

23. $\left(-\sqrt{2}, \sqrt{2}\right)$

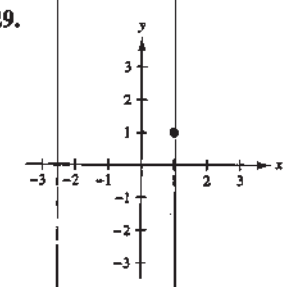
25. $(-1.204, -4.336)$

27.



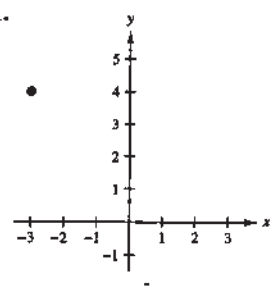
$(7, \pi), (-7, 0)$

29.



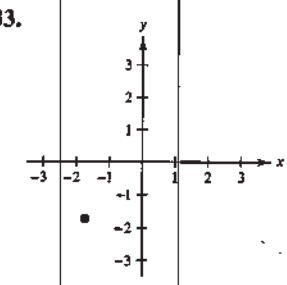
$\left(\sqrt{2}, \frac{\pi}{4}\right), \left(-\sqrt{2}, \frac{5\pi}{4}\right)$

31.



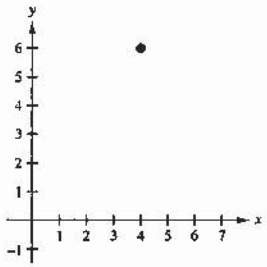
$(5, 2.214), (-5, 5.356)$

33.



$\left(\sqrt{6}, \frac{5\pi}{4}\right), \left(-\sqrt{6}, \frac{\pi}{4}\right)$

35.



$(2\sqrt{13}, 0.983)$, $(-2\sqrt{13}, 4.124)$

37. $(\sqrt{13}, -0.588)$ **39.** $(\sqrt{7}, 0.857)$ **41.** $(\frac{17}{6}, 0.490)$

43. $\left(5, -\frac{\pi}{2}\right)$ **45.** (a) $r = 7$ (b) $r = a$

47. (a) $r = 2a\cos\theta$ (b) $r = 2a\sin\theta$

49. (a) $r = 12\sec\theta$ (b) $r = a\sec\theta$

51. (a) $r^2 = 4\csc\theta\sec\theta = 8\csc 2\theta$

(b) $r^2 = \frac{1}{2}\csc\theta\sec\theta = \csc 2\theta$

53. (a) $r = \tan^2\theta\sec\theta$ (b) $r = \cot^2\theta\csc\theta$

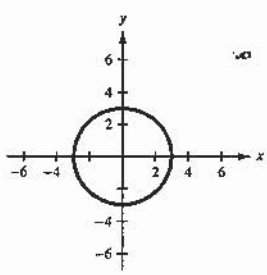
55. $x^2 + y^2 - 4y = 0$ **57.** $\sqrt{3}x - 3y = 0$

59. $x^2 + y^2 = 16$ **61.** $y = -3$ **63.** $(x^2 + y^2)^3 = x^2$

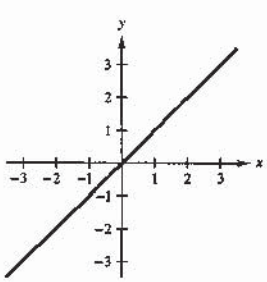
65. $(x^2 + y^2)^2 = 6x^2y - 2y^3$ **67.** $y^2 = 2x + 1$

69. $4x^2 - 5y^2 - 36y - 36 = 0$

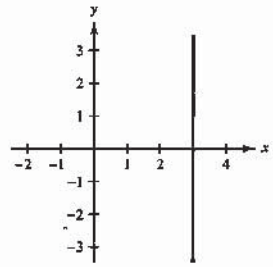
71. The graph is a circle centered at the origin with a radius of 3; $x^2 + y^2 = 9$.



73. The graph consists of all points on the line that make an angle of $\pi/4$ with the positive x-axis; $x - y = 0$.



75. The graph is not evident by simple inspection; $x - 3 = 0$.



77. True. Because r is a directed distance, (r, θ) can be represented by $(-r, \theta \pm (2n + 1)\pi)$, so $|r| = |-r|$.

79. (a) Answers will vary.

(b) The points lie on a line.

$d = \sqrt{r_1^2 + r_2^2 - 2r_1r_2} = |r_1 - r_2|$

(c) $d = \sqrt{r_1^2 + r_2^2}$ (Pythagorean Theorem)

(d) Answers will vary. The distance formulas should give the same results.

81. $(2, 3)$ **83.** $\left(\frac{8}{7}, \frac{88}{35}, \frac{8}{5}\right)$ **85.** 175,000

87. $-101{,}376$

Section 10.7 *(page 752)*

1. Rose curve **3.** Circle **5.** Rose curve

7. Polar axis **9.** $\theta = \frac{\pi}{2}$ **11.** $\theta = \frac{\pi}{2}$

13. $\theta = \frac{\pi}{2}$, polar axis, pole **15.** Pole

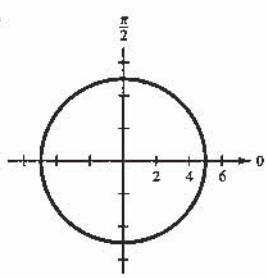
17. Maximum: $|r| = 20$ when $\theta = \frac{3\pi}{2}$

Zero: $r = 0$ when $\theta = \frac{\pi}{2}$

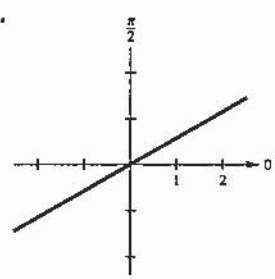
19. Maximum: $|r| = 4$ when $\theta = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi$

Zeros: $r = 0$ when $\theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$

21.



23.



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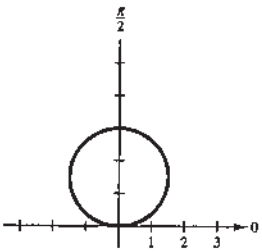
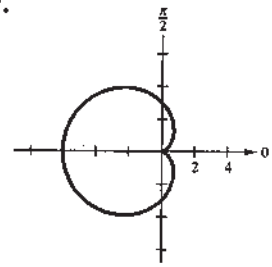


27.



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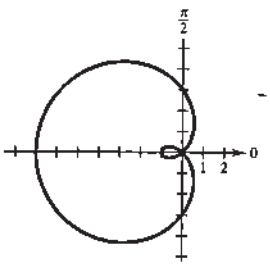
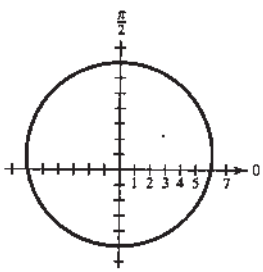


31.



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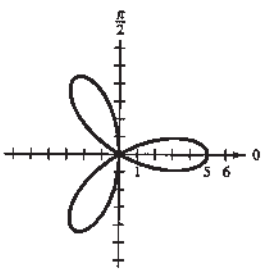
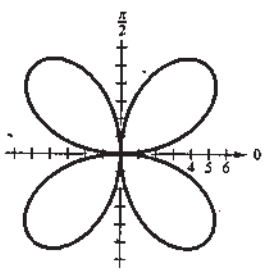


35.



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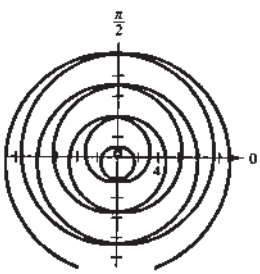

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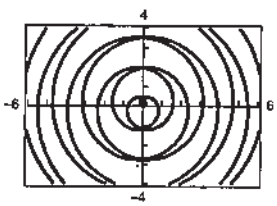
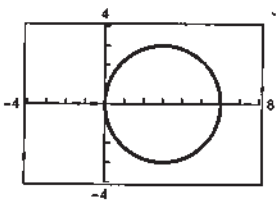


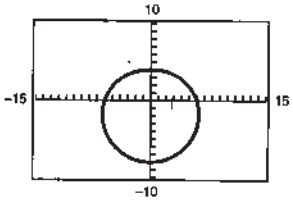
$-10\pi \le \theta < 10\pi$

41.



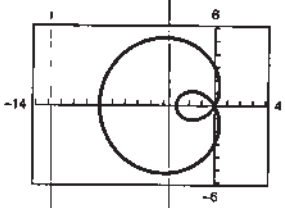
$0 \le \theta < \pi$

43.



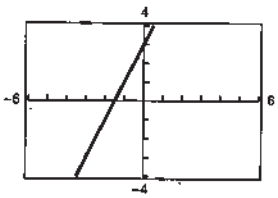
$0 \le \theta < 2\pi$

45.



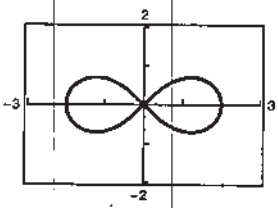
$0 \le \theta < 2\pi$

47.



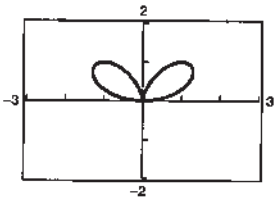
$0 \le \theta < \dfrac{\pi}{2}$

49.



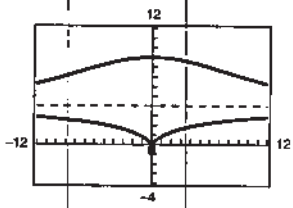
$-2\pi \le \theta < 2\pi$

51.



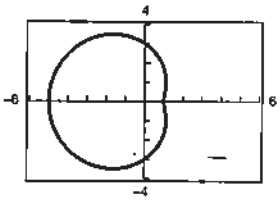
$0 \le \theta < \pi$

53.



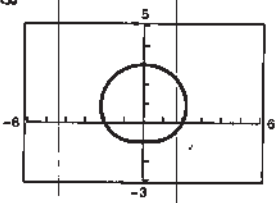
$0 \le \theta < 2\pi$

55.



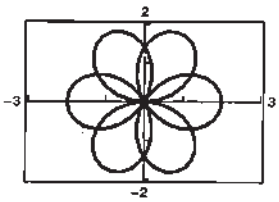
$0 \le \theta < 2\pi$

57.



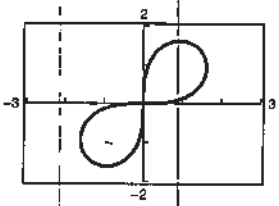
$0 \le \theta < 2\pi$

59.



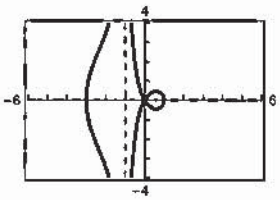
$0 \le \theta < 4\pi$

61.



$0 \le \theta < \dfrac{\pi}{2}$

63.



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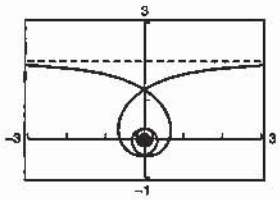
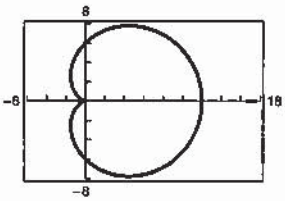


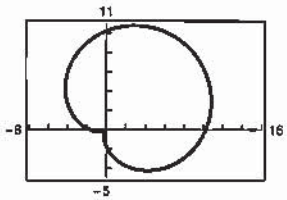
67. False. When the coordinates are substituted into the equation you get $6 = 4$, which is a false statement.

69. False. The rose curve will have 5 petals.

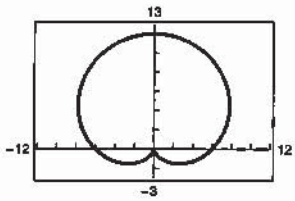
71. (a)



(b)



(c)



$r = 6(1 + \sin\theta)$

ϕ rotates the graph around the pole.

73. (a), (b), and (c) Answers will vary.

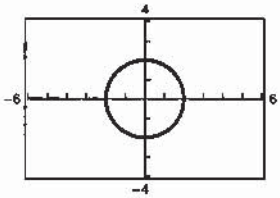
75. (a) $r = 4\sin\left(\theta - \frac{\pi}{6}\right)\cos\left(\theta - \frac{\pi}{6}\right)$

(b) $r = -4\sin\theta\cos\theta$

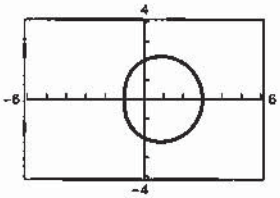
(c) $r = 4\sin\left(\theta - \frac{2\pi}{3}\right)\cos\left(\theta - \frac{2\pi}{3}\right)$

(d) $r = 4\sin\theta\cos\theta$

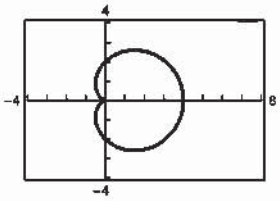
77.



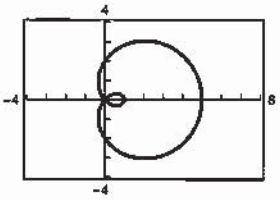
$k = 0$: circle



$k = 1$: convex limaçon



$k = 2$: cardioid



$k = 3$: limaçon with inner loop

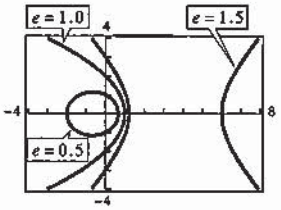
79. 2, 5, 8, 11, 14; $d = 3$; $a_n = -1 + 3n$

81. 150, 132, 114, 96, 78; $d = -18$; $a_n = 168 - 18n$

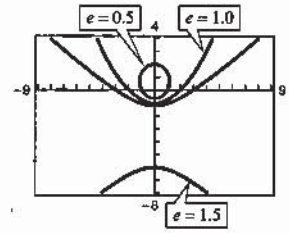
83. 840 **85.** 7860 **87.** $\frac{8}{7}$

Section 10.8 *(page 758)*

1.



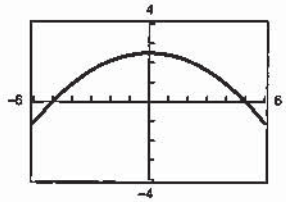
3.



5. (b) **7.** (d) **9.** Parabola **11.** Ellipse

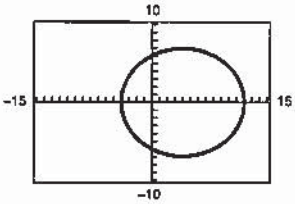
13. Ellipse **15.** Hyperbola **17.** Hyperbola

19.



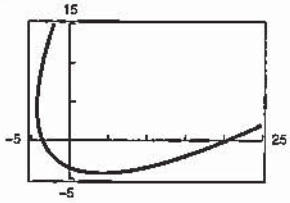
Parabola

21.

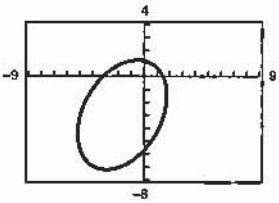


Ellipse

23.



25.



27. $r = \dfrac{1}{1 - \cos\theta}$ **29.** $r = \dfrac{1}{2 + \sin\theta}$

31. $r = \dfrac{2}{1 + 2\cos\theta}$ **33.** $r = \dfrac{2}{1 - \sin\theta}$

35. $r = \dfrac{16}{5 + 3\cos\theta}$ **37.** $r = \dfrac{9}{4 - 5\sin\theta}$

39. Answers will vary.

41. $r = \dfrac{9.2931 \times 10^7}{1 - 0.0167\cos\theta}$

Perihelion: 9.1405×10^7 miles

Aphelion: 9.4509×10^7 miles

43. $r = \dfrac{7977.2}{1 - 0.937\cos\theta}$

11,008 miles

45. False. The equation can be rewritten as

$$r = \frac{-4/3}{1 + \sin\theta}.$$

Because ep is negative, you know that e is positive and p is negative and that p represents the distance between the pole and the directrix, so the directrix has to be below the pole.

47. Answers will vary. **49.** $r^2 = \dfrac{24{,}336}{169 - 25\cos^2\theta}$

51. 60 **53.** 90,720 **55.** 220 **57.** 720

Review Exercises *(page 761)*

1. Hyperbola **3.** $(x - 4)^2 = -8(y - 2)$

5. $(y - 2)^2 = 12x$ **7.** $2x + y - 2 = 0$; $(1, 0)$

9. About 19.6 meters **11.** $\dfrac{(x-2)^2}{25} + \dfrac{y^2}{21} = 1$

13. $\dfrac{(x-2)^2}{4} + (y-1)^2 = 1$ **15.** 3 feet atop the pillars

17. Center: $(1, -4)$

Vertices:

$(1, 0), (1, -8)$

Foci: $\left(1, -4 \pm \sqrt{7}\right)$

Eccentricity: $\dfrac{\sqrt{7}}{4}$

19. Center: $(-2, 1)$

Vertices:

$(-2, 11), (-2, -9)$

Foci: $\left(-2, 1 \pm \sqrt{19}\right)$

Eccentricity: $\dfrac{\sqrt{19}}{10}$

21. $\dfrac{(x+2)^2}{64} - \dfrac{(y-3)^2}{36} = 1$ **23.** $\dfrac{5(x-4)^2}{16} - \dfrac{5y^2}{64} = 1$

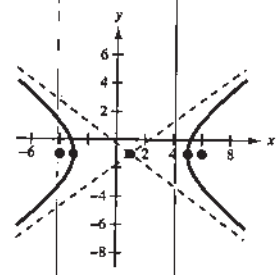
25. Center: $(1, -1)$

Vertices: $(5, -1), (-3, -1)$

Foci: $(6, -1), (-4, -1)$

Asymptotes:

$y = -1 \pm \frac{3}{4}(x - 1)$



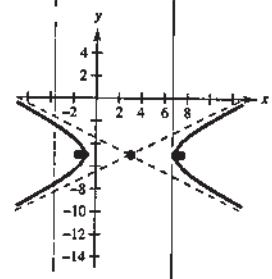
27. Center: $(3, -5)$

Vertices: $(7, -5), (-1, -5)$

Foci: $\left(3 \pm 2\sqrt{5}, -5\right)$

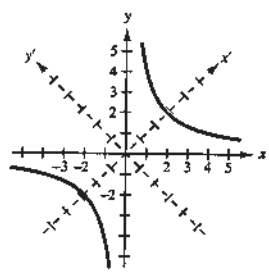
Asymptotes:

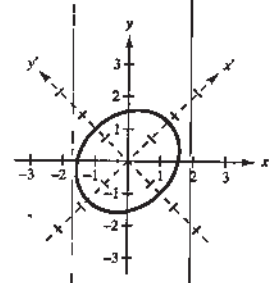
$y = -5 \pm \frac{1}{2}(x - 3)$



29. 72 miles **31.** Ellipse

33. $\dfrac{(x')^2}{8} - \dfrac{(y')^2}{8} = 1$ **35.** $\dfrac{(x')^2}{3} + \dfrac{(y')^2}{2} = 1$

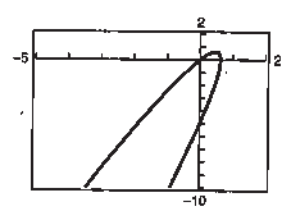




37. (a) Parabola

(b) $y = \dfrac{8x - 5 \pm \sqrt{(8x-5)^2 - 4(16x^2 - 10x)}}{2}$

(c)

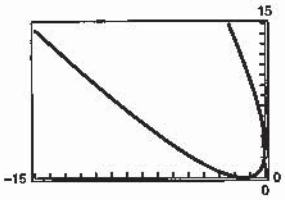


39. (a) Parabola

(b) $y =$

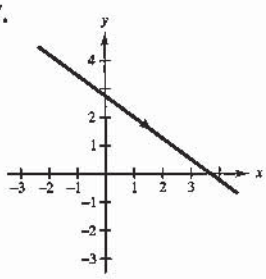
$$\frac{-\left(2x - 2\sqrt{2}\right) \pm \sqrt{\left(2x - 2\sqrt{2}\right)^2 - 4\left(x^2 + 2\sqrt{2}x + 2\right)}}{2}$$

(c)



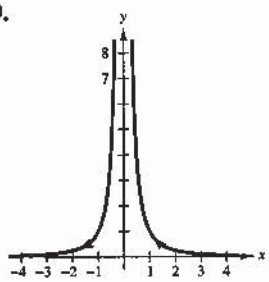
41. $(-10, 12)$ **43.** $x = 3, y = 0$ **45.** $x = \frac{3\sqrt{3}}{2}, y = \frac{1}{2}$

47.



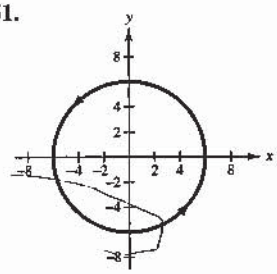
$y = -\frac{3}{4}x + \frac{11}{4}$

49.



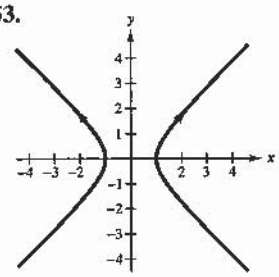
$y = \frac{1}{x^2}$

51.



$x^2 + y^2 = 36$

53.

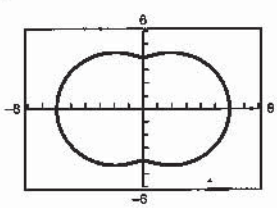


$x^2 - y^2 = 1$

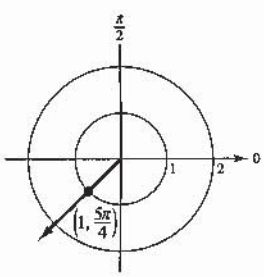
55. $x = -3 + 4\cos\theta$

$y = 4 + 3\sin\theta$

57.

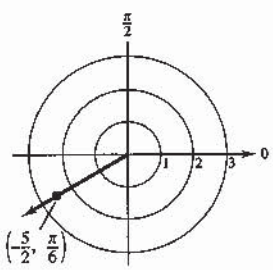


59.



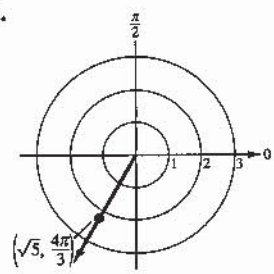
$\left(1, -\frac{3\pi}{4}\right), \left(-1, \frac{\pi}{4}\right), \left(-1, -\frac{7\pi}{4}\right)$

61.



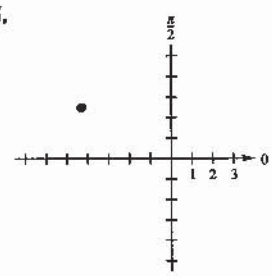
$\left(-\frac{5}{2}, -\frac{11\pi}{6}\right), \left(\frac{5}{2}, \frac{7\pi}{6}\right), \left(\frac{5}{2}, -\frac{5\pi}{6}\right)$

63.



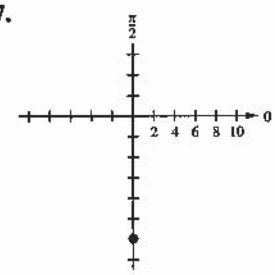
$\left(\sqrt{5}, -\frac{2\pi}{3}\right), \left(-\sqrt{5}, \frac{\pi}{3}\right), \left(-\sqrt{5}, -\frac{5\pi}{3}\right)$

65.



$\left(-\frac{5\sqrt{3}}{2}, \frac{5}{2}\right)$

67.



$(0, -12)$

69. $\left(-9, \frac{\pi}{2}\right), \left(9, \frac{3\pi}{2}\right)$ **71.** $\left(-5\sqrt{2}, \frac{3\pi}{4}\right), \left(5\sqrt{2}, \frac{7\pi}{4}\right)$

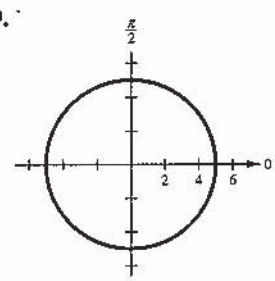
73. $x^2 + y^2 = 25$ **75.** $x^2 + y^2 = 3x$

77. $(x^2 + y^2)^2 - x^2 + y^2 = 0$

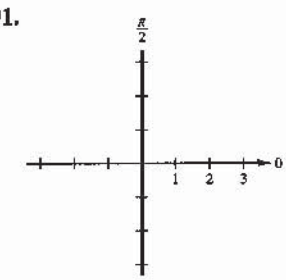
79. $4x^2 + 3y^2 - 4y - 4 = 0$ **81.** $r = 3$

83. $r = 6\csc\theta$ **85.** $r = 4\cos\theta$ **87.** $r^2 = 5\sec\theta\csc\theta$

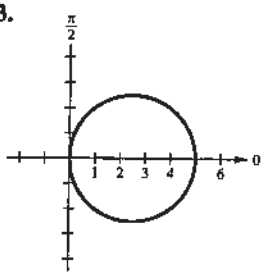
89.



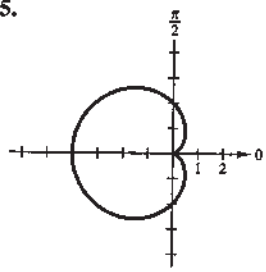
91.



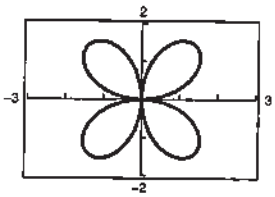
93.



95.

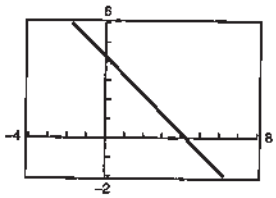


97.



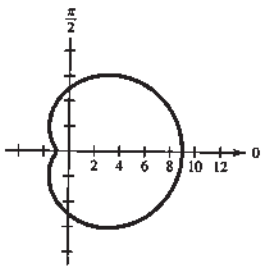
Rose curve with four petals

99.



Straight line, $y = 4 - x$

101.

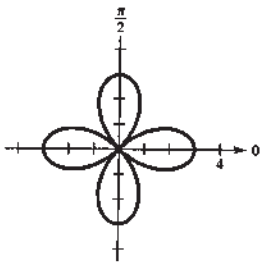


Symmetry: Polar axis

Maximum: $|r| = 9$ when $\theta = 0, 2\pi$

Zeros of r: None

103.

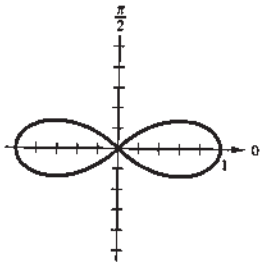


Symmetry: Pole, polar axis, and the line $\theta = \frac{\pi}{2}$

Maximum: $|r| = 3$ when $\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$

Zeros of r: when $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

105.

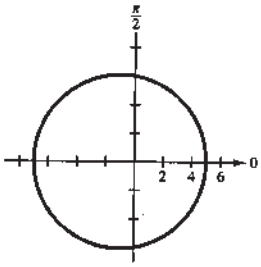


Symmetry: Pole, polar axis, and the line $\theta = \frac{\pi}{2}$

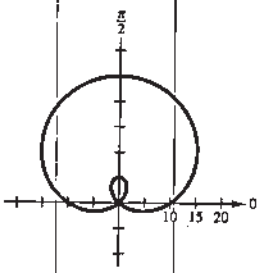
Maximum: $|r| = 1$ when $\theta = 0, \pi, 2\pi$

Zeros of r: when $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

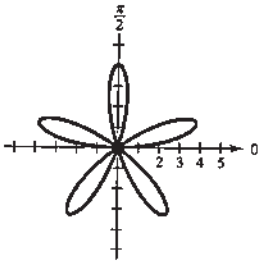
107. Limaçon



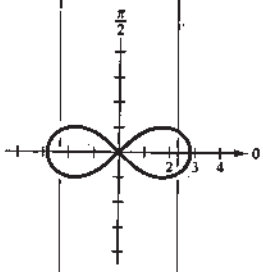
109. Limaçon



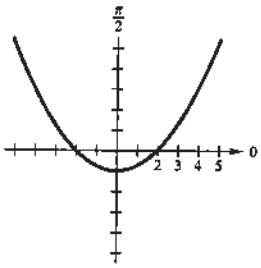
111. Rose curve



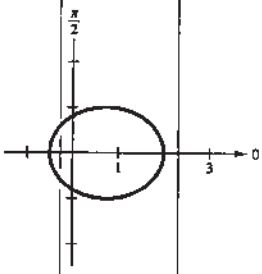
113. Lemniscate



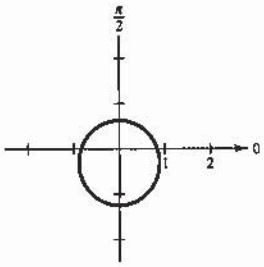
115. Parabola



117. Ellipse



119. Ellipse



121. $r = 16 \sin \theta$ **123.** $r = \dfrac{5}{3 - 2\cos\theta}$

125. $r = \dfrac{1.512}{1 - 0.092\cos\theta}$

Perihelion: 1.3847 astronomical units

Aphelion: 1.6653 astronomical units

127. False. The following are two sets of parametric equations for the line.

$x = t, \quad y = 3 - 2t$

$x = 3t, \quad y = 3 - 6t$

129. (a) Vertical translation

(b) Horizontal translation

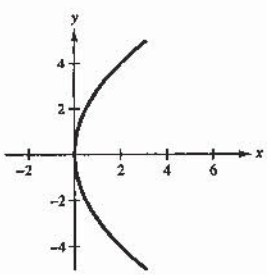
(c) Reflection in the y-axis

(d) Parabola opens more slowly.

131. 5; The ellipse becomes more circular and approaches a circle of radius 5.

Chapter Test *(page 766)*

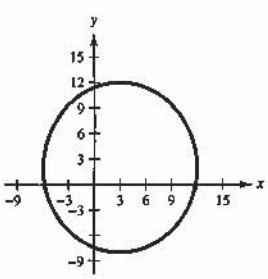
1.



Vertex: $(0, 0)$

Focus: $(2, 0)$

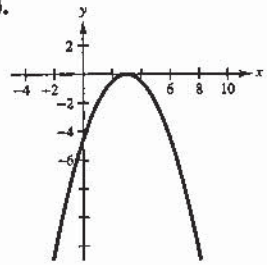
2.



Vertices: $(3, 12), (3, -8)$

Foci: $(3, 2 \pm \sqrt{19})$

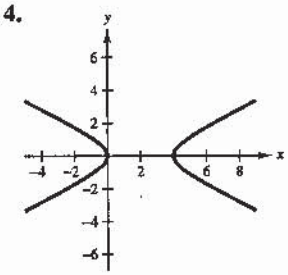
3.



Vertex: $(3, 0)$

Focus: $\left(3, -\frac{1}{2}\right)$

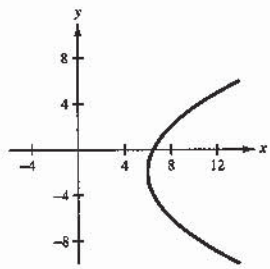
4.



Vertices: $(0, 0), (4, 0)$

Foci: $(2 \pm \sqrt{5}, 0)$

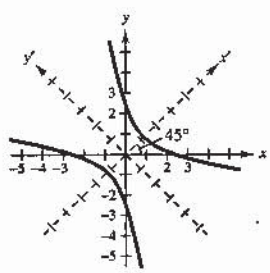
5. $(y + 2)^2 = 8(x - 6)$



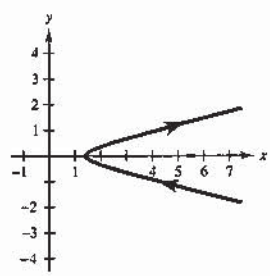
6. $\dfrac{(x+6)^2}{16} + \dfrac{(y-3)^2}{49} = 1$ **7.** $\dfrac{y^2}{9} - \dfrac{x^2}{4} = 1$

8. (a) $45°$ **9.** No solution

(b)



10.



$y = \dfrac{\pm\sqrt{x^2 - 2}}{4}, \; x \geq \sqrt{2}$

11. $x = t + 4, \; y = \dfrac{t}{4} - 4$

$x = 2t, \; y = \dfrac{1}{2}t - 5$

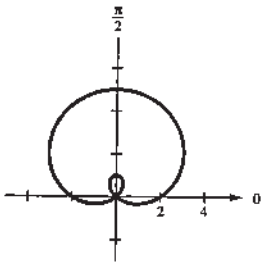
(Solutions are not unique.)

12. $(-7, 7\sqrt{3})$

13. $\left(2\sqrt{2}, \frac{5\pi}{4}\right), \left(-2\sqrt{2}, \frac{\pi}{4}\right), \left(2\sqrt{2}, -\frac{3\pi}{4}\right)$

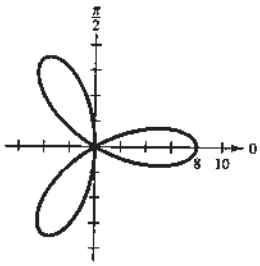
14. $r = 12 \sin \theta$

15.



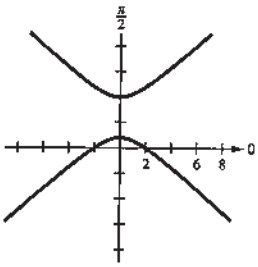
Limaçon

16.



Rose curve

17.



Hyperbola

18. $r = \dfrac{1}{1 + \frac{1}{4}\sin \theta}$

19. $r = \dfrac{34{,}478{,}231}{1 + 0.2056 \sin \theta}$

Perihelion: 28,598,400 miles

Aphelion: 43,401,600 miles

Cumulative Test for Chapters 7–10
(page 767)

1. $(4, -3)$ **2.** $\left(\frac{3}{5}, -4, -\frac{1}{5}\right)$

3. $\begin{bmatrix} -7 & -10 & -16 \\ -6 & 18 & 9 \\ -12 & 16 & 7 \end{bmatrix}$ **4.** $\begin{bmatrix} -18 & 15 & -14 \\ 28 & 11 & 34 \\ -20 & 52 & -1 \end{bmatrix}$

5. $\begin{bmatrix} 3 & -31 & 2 \\ 22 & 18 & 6 \\ 52 & -40 & 14 \end{bmatrix}$ **6.** $\begin{bmatrix} 5 & 36 & 31 \\ -36 & 12 & -36 \\ 16 & 0 & 18 \end{bmatrix}$

7. (a) $\begin{bmatrix} -175 & 37 & -13 \\ 95 & -20 & 7 \\ 14 & -3 & 1 \end{bmatrix}$ (b) 1

8. $(4, -2, 1)$ **9.** 22 square units **10.** 1110

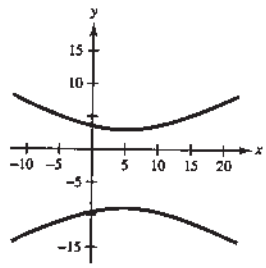
11. 135 **12.** $\frac{47}{52}$ **13.** ≈ 34.480 **14.** 80 **15.** 3685

16. Answers will vary.

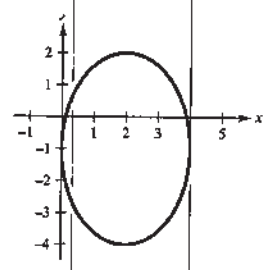
17. $x^6 - 12x^5y + 60x^4y^2 - 160x^3y^3 + 240x^2y^4 - 192xy^5 + 64y^6$

18. 151,200

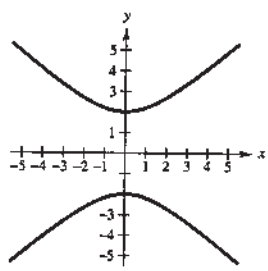
19. Hyperbola



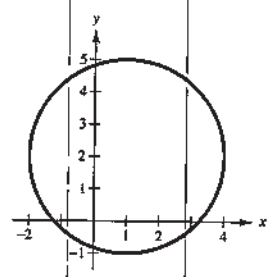
20. Ellipse



21. Hyperbola



22. Circle

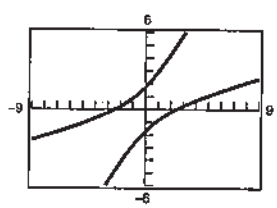


23. $(x - 2)^2 = -\frac{4}{3}(y - 3)$

24. $\dfrac{(x-1)^2}{25} + \dfrac{(y-4)^2}{4} = 1$ **25.** $\dfrac{(y+4)^2}{4} - \dfrac{x^2}{16/3} = 1$

26. $\dfrac{(y-2)^2}{4/5} - \dfrac{x^2}{16/5} = 1$

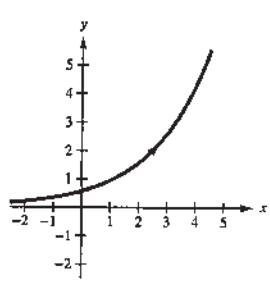
27.



$\theta \approx 38°$

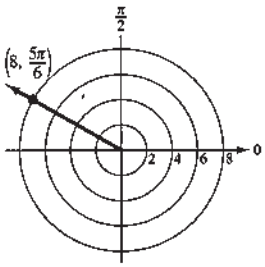
28. $(0, 6), (12, 6)$

29.



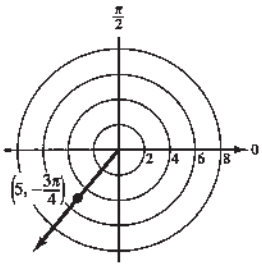
$y = \frac{1}{2}e^{x/2}$

30.



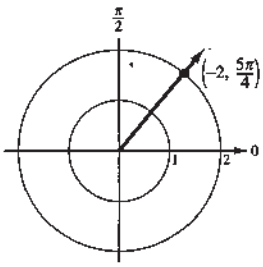
$\left(8, -\frac{7\pi}{6}\right), \left(-8, -\frac{\pi}{6}\right), \left(-8, \frac{11\pi}{6}\right)$

31.



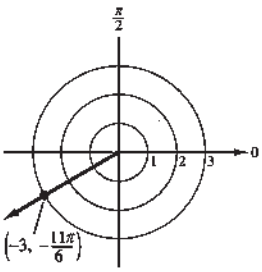
$\left(5, \frac{5\pi}{4}\right), \left(-5, -\frac{7\pi}{4}\right), \left(-5, \frac{\pi}{4}\right)$

32.



$\left(-2, -\frac{3\pi}{4}\right), \left(2, -\frac{7\pi}{4}\right), \left(2, \frac{\pi}{4}\right)$

33.

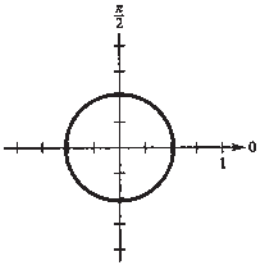


$\left(-3, \frac{\pi}{6}\right), \left(3, -\frac{5\pi}{6}\right), \left(3, \frac{7\pi}{6}\right)$

34. $r = \dfrac{5}{8\cos\theta + 3\sin\theta}$

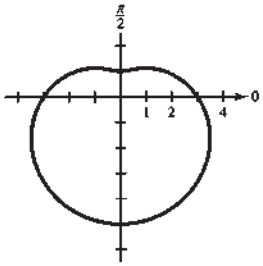
35. $9x^2 - 16y^2 + 20x + 4 = 0$

36.



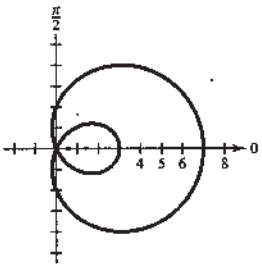
Circle

37.



Dimpled limaçon

38.



Limaçon

39. $r = \dfrac{14}{4 - 3\cos\theta}$ **40.** 1365 **41.** 156,238,908

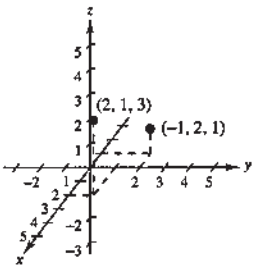
42. $\frac{1}{4}$

Chapter 11

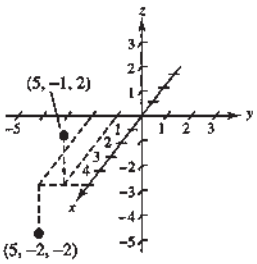
Section 11.1 *(page 775)*

1. A: $(-1, 4, 3)$, B: $(1, 3, -2)$, C: $(-3, 0, -2)$

3.



5.



7. $(-3, 3, 5)$ **9.** $(12, 0, 0)$ **11.** Octant IV

13. Octants I, II, III, and IV **15.** Octants II, IV, VI, and VIII

17. $\sqrt{29}$ units **19.** $2\sqrt{38}$ units **21.** $\sqrt{110}$ units

23. $(2\sqrt{5})^2 + 3^2 = (\sqrt{29})^2$

25. $6, 6, 2\sqrt{10}$; Isosceles triangle **27.** $(0, -1, 7)$

29. $(1, 0, 5.5)$ **31.** $(2.5, 2, 6)$

33. $(x - 3)^2 + (y - 2)^2 + (z - 4)^2 = 16$

35. $x^2 + (y - 4)^2 + (z - 3)^2 = 9$

37. $(x+3)^2 + (y-7)^2 + (z-5)^2 = 25$

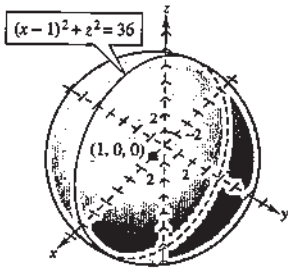
39. $\left(x - \frac{3}{2}\right)^2 + y^2 + (z-3)^2 = \frac{45}{4}$

41. Center: $(2, -1, 3)$; Radius: 2

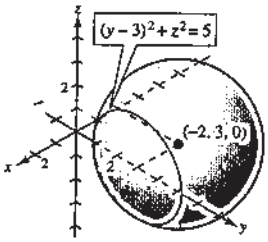
43. Center: $(-2, 0, 4)$; Radius: 1

45. Center: $\left(1, \frac{1}{3}, 4\right)$; Radius: 3

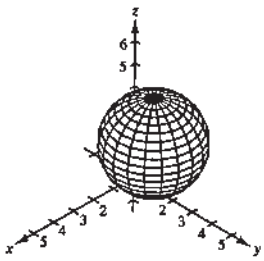
47.



49.



51.



53. $(3, 3, 3)$

55. False. z is the directed distance from the xy-plane to P.

57. 0; 0; 0 **59.** A straight line or the xy-plane

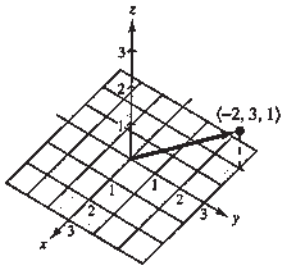
61. $(7, 16, 12)$ **63.** $(x+2)^2 = -20(y-5)$

65. $\dfrac{x^2}{45/4} + \dfrac{(y-3)^2}{81/4} = 1$ **67.** $\dfrac{(y-5)^2}{16} - \dfrac{(x-3)^2}{9} = 1$

Section 11.2 *(page 782)*

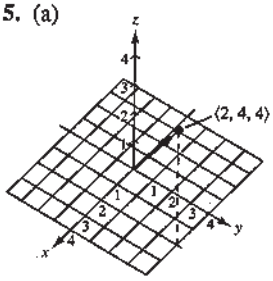
1. (a) $\langle -2, 3, 1\rangle$

(b)

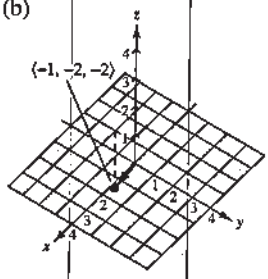


3. (a) $\langle 4, 4, 4\rangle$ (b) $4\sqrt{3}$ (c) $\left\langle \dfrac{\sqrt{3}}{3}, \dfrac{\sqrt{3}}{3}, \dfrac{\sqrt{3}}{3} \right\rangle$

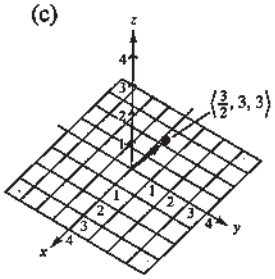
5. (a)



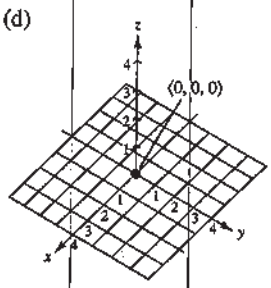
(b)



(c)



(d)



7. $\mathbf{z} = \langle -3, 7, 6\rangle$ **9.** $\mathbf{z} = \left\langle \frac{1}{2}, 6, \frac{3}{2}\right\rangle$ **11.** $\sqrt{33}$

13. $\sqrt{74}$ **15.** $\sqrt{34}$

17. (a) $\dfrac{1}{\sqrt{74}}(8\mathbf{i} + 3\mathbf{j} - \mathbf{k})$ (b) $-\dfrac{1}{\sqrt{74}}(8\mathbf{i} + 3\mathbf{j} - \mathbf{k})$

19. $\langle -26, 0, 48\rangle$ **21.** $\frac{1}{2}\sqrt{305}$

23. $\left\langle \dfrac{2\sqrt{13}}{13}, \dfrac{9\sqrt{13}}{65}, -\dfrac{12\sqrt{13}}{65} \right\rangle$ **25.** -4 **27.** 0

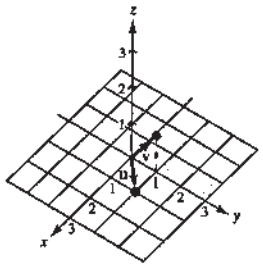
29. $\approx 124.45°$ **31.** $\approx 109.92°$ **33.** Parallel

35. Orthogonal **37.** Not collinear **39.** Collinear

41. $(3, 1, 7)$ **43.** $\left(6, \dfrac{5}{2}, -\dfrac{7}{4}\right)$ **45.** $\pm\dfrac{3\sqrt{14}}{14}$

47. $\langle 0, 2\sqrt{2}, 2\sqrt{2}\rangle$ **49.** 10.91 pounds **51.** True

53. (a)



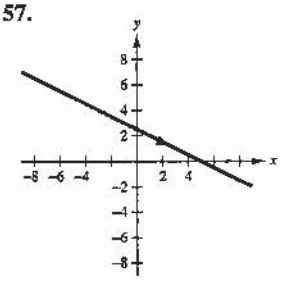
(b) Answers will vary.

(c) $a = b = 1$

(d) Answers will vary.

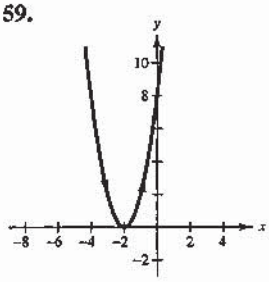
55. The angle between $\mathbf{u}$ and $\mathbf{v}$ is an obtuse angle.

57.



$y = -\dfrac{x+5}{2}$

59.

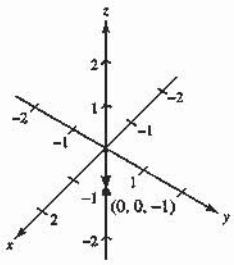


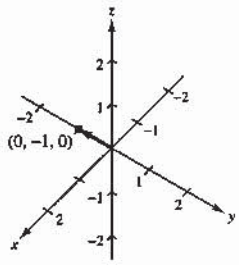
$y = 2(x+2)^2$

61. -77

Section 11.3 *(page 789)*

1. $-\mathbf{k}$ **3.** $-\mathbf{j}$





5. $\langle 0, 0, 14\rangle$ **7.** $\langle -3, 5, 23\rangle$ **9.** $-7\mathbf{i} + 13\mathbf{j} + 16\mathbf{k}$

11. $-18\mathbf{i} - 6\mathbf{j}$ **13.** $\langle 10, -2, -4\rangle$

15. $-3\mathbf{i} - \frac{11}{3}\mathbf{j} - \frac{1}{3}\mathbf{k}$ **17.** $-\mathbf{i} - 2\mathbf{j} - \mathbf{k}$

19. $\dfrac{\sqrt{19}}{19}(\mathbf{i} - 3\mathbf{j} + 3\mathbf{k})$ **21.** $\dfrac{\sqrt{38}}{38}(-2\mathbf{i} + 5\mathbf{j} - 3\mathbf{k})$

23. $\dfrac{\sqrt{2}}{2}(\mathbf{i} - \mathbf{j})$ **25.** 1 **27.** $\sqrt{806}$ **29.** 14

31. (a) $\overrightarrow{AB} = \langle 1, 2, -2\rangle$ and is parallel to $\overrightarrow{DC} = \langle 1, 2, -2\rangle$.
$\overrightarrow{AD} = \langle -3, 4, 4\rangle$ and is parallel to $\overrightarrow{BC} = \langle -3, 4, 4\rangle$.
(b) Area is $\|\overrightarrow{AB} \times \overrightarrow{AD}\| = 6\sqrt{10}$.
(c) $\theta \approx 81.02°$; Parallelogram is not a rectangle.

33. $\sqrt{349}$ **35.** $\frac{1}{2}\sqrt{4290}$ **37.** -16 **39.** 2

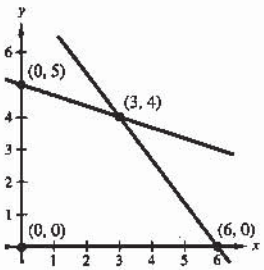
41. 2 **43.** 12 **45.** 84

47. $10 \cos 40° \approx 7.66$ foot-pounds **49.** True

51. and 53. Answers will vary. **55.** $-\sqrt{3}$ **57.** $-\dfrac{\sqrt{3}}{2}$

59. $-\dfrac{\sqrt{3}}{2}$ **61.** $\sqrt{3}$

63.



Maximum value of z is 46, at $x = 3$ and $y = 4$.
Minimum value of z is 0, at $x = 0$ and $y = 0$.

Section 11.4 *(page 798)*

1. (a) $x = -1 - 2t,\ y = 4 + 4t,\ z = t$
(b) $\dfrac{x+1}{-2} = \dfrac{y-4}{4} = z$

3. (a) $x = -4 + 3t,\ y = 1 + 8t,\ z = -6t$
(b) $\dfrac{x+4}{3} = \dfrac{y-1}{8} = \dfrac{z}{-6}$

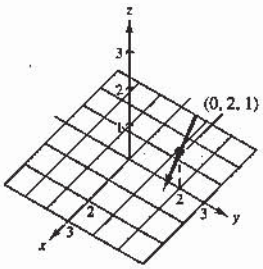
5. (a) $x = 2 + 2t,\ y = -3 - 3t,\ z = 5 + t$
(b) $\dfrac{x-2}{2} = \dfrac{y+3}{-3} = z - 5$

7. (a) $x = 6 - 4t,\ y = t,\ z = 3 + 5t$
(b) $\dfrac{x-6}{-4} = y = \dfrac{z-3}{5}$

9. (a) $x = -3 + 4t,\ y = 8 - 10t,\ z = 15 + t$
(b) $\dfrac{x+3}{4} = \dfrac{y-8}{-10} = z - 15$

11. (b), (c)

13.



15. $y - 4 = 0$

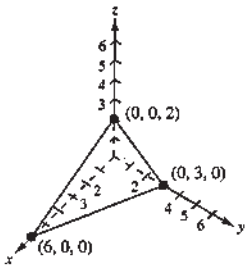
17. $-2x + y - 2z + 10 = 0$ **19.** $-x - 2y + z + 2 = 0$

21. $-3y + z = 0$ **23.** $-5x + 6y + z + 8 = 0$

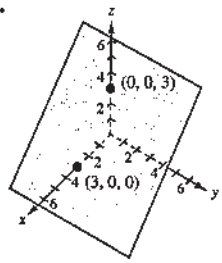
25. $y - 5 = 0$ **27.** $-2x - 4y + 5z + 8 = 0$

29. Parallel **31.** Orthogonal

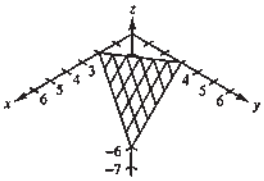
33.



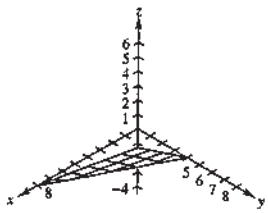
35.



37.



39.



41. $\dfrac{12}{\sqrt{14}}$ **43.** $\dfrac{4}{\sqrt{6}}$

45. (a) 60.7° (b) $x = -t + 2,\ y = 8t,\ z = 7t$

47. (a) 77.8° (b) $x = 6t + 1,\ y = t,\ z = 7t + 1$

49. 88.5°

51. False. Lines that do not intersect and are not in the same plane may not be parallel.

53. True

55. (a) Sphere: $(x - 4)^2 + (y + 1)^2 + (z - 1)^2 = 4$

(b) Two planes: $4x - 3y + z = 10 \pm 2\sqrt{26}$

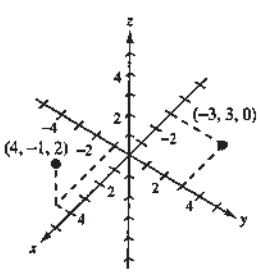
57. $y = -x$ **59.** $3x^2 + 4y^2 - 2x - 1 = 0$

61. $r = 4\cos\theta$ **63.** $r = 3\sec\theta$

65. $r = \dfrac{4}{5\cos\theta - 6\sin\theta}$

Review Exercises *(page 801)*

1.



3. $(-5, 4, 0)$ **5.** $\sqrt{41}$

7. $\sqrt{29}, \sqrt{38}, \sqrt{67}$

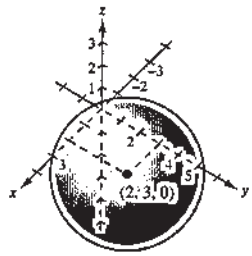
$\left(\sqrt{29}\right)^2 + \left(\sqrt{38}\right)^2 = \left(\sqrt{67}\right)^2$

9. $\left(\frac{13}{2}, 2, 5\right)$ **11.** $(1, 2, -9)$

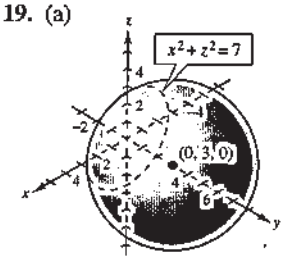
13. $(x - 2)^2 + (y - 3)^2 + (z - 5)^2 = 1$

15. $(x - 1)^2 + (y - 5)^2 + (z - 2)^2 = 36$

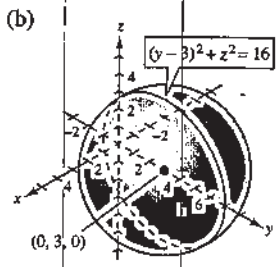
17. Center: $(2, 3, 0)$, $r = 3$



19. (a)



(b)



21. $\langle 1, 4, -4\rangle, \sqrt{33}$ **23.** $\langle -10, 6, 7\rangle, \sqrt{185}$ **25.** 2

27. 1 **29.** 90° **31.** Parallel **33.** Answers will vary.

35. *A*: 159.1 pounds of tension

B: 115.6 pounds of tension

C: 115.6 pounds of tension

37. $\langle -10, 0, -10\rangle$

39. $-\dfrac{71}{\sqrt{7602}}\mathbf{i} - \dfrac{44}{\sqrt{7602}}\mathbf{j} + \dfrac{25}{\sqrt{7602}}\mathbf{k}$

41. Area $= \sqrt{172} \approx 13.11$ **43.** 75

45. (a) $x = 5t,\ y = -10 + 20t,\ z = 3 - 3t$

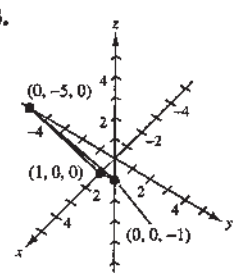
(b) $\dfrac{x}{5} = \dfrac{y + 10}{20} = \dfrac{z - 3}{-3}$

47. (a) $x = 3 + t,\ y = 2 + t,\ z = 1 + t$

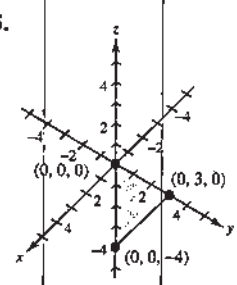
(b) $x - 3 = y - 2 = z - 1$

49. $-2y + 5z - 14 = 0$ **51.** $x + y + z - 6 = 0$

53.



55.

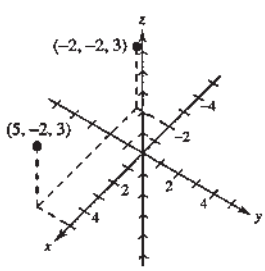


57. $\dfrac{1}{\sqrt{6}}$ **59.** $\dfrac{12}{\sqrt{14}}$ **61.** True

63. and 65. Answers will vary.

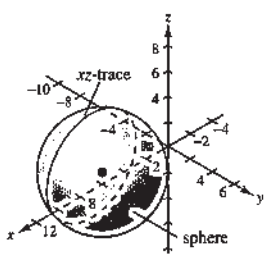
Chapter Test *(page 804)*

1.



2. No. $\left(\sqrt{76}\right)^2 + \left(\sqrt{102}\right)^2 \neq \left(\sqrt{194}\right)^2$ **3.** $(7, 1, 2)$

4.



$(x - 7)^2 + (y - 1)^2 + (z - 2)^2 = 19$

5. $\mathbf{u} = \langle -2, 6, -6 \rangle$, $\mathbf{v} = \langle -12, 5, -5 \rangle$

6. (a) $\sqrt{194}$ (b) 84 (c) $\langle 0, 62, 62 \rangle$ **7.** 46.23°

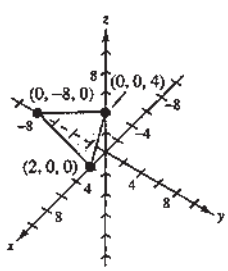
8. (a) $x = 8 - 2t,\ y = -2 + 6t,\ z = 5 - 6t$

(b) $\dfrac{x - 8}{-2} = \dfrac{y + 2}{6} = \dfrac{z - 5}{-6}$

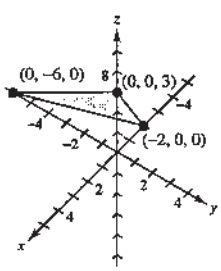
9. $y + z - 3 = 0$ **10.** Neither **11.** Orthogonal

12. 200

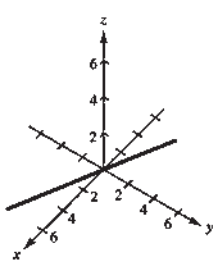
13.



14.



15. $\dfrac{4\sqrt{14}}{7}$ **16.**

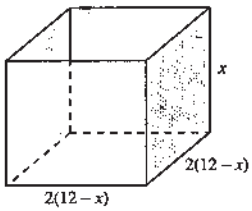


17. $6\sqrt{3}$

Chapter 12

Section 12.1 *(page 813)*

1. (a)



(b) $V = lwh$

$= 2(12 - x) \cdot 2(12 - x) \cdot x$

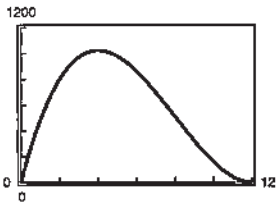
$= 4x(12 - x)^2$

(c)

x	3	3.5	3.9	4	4.1	4.5	5
V	972	1011.5	1023.5	1024	1023.5	1012.5	980

$\lim\limits_{x \to 4} V = 1024$

(d)



3.

x	2.9	2.99	2.999	3
$f(x)$	−4.7	−4.97	−4.997	−5

x	3.001	3.01	3.1
$f(x)$	−5.003	−5.03	−5.3

-5; yes

5.

x	2.9	2.99	2.999	3
$f(x)$	0.1695	0.1669	0.1667	Error

x	3.001	3.01	3.1
$f(x)$	0.1666	0.1664	0.1639

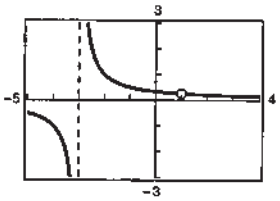
$\frac{1}{6}$; no

7.

x	0.9	0.99	0.999	1
$f(x)$	0.2564	0.2506	0.2501	Error

x	1.001	1.01	1.1
$f(x)$	0.2499	0.2494	0.2439

$\frac{1}{4}$;

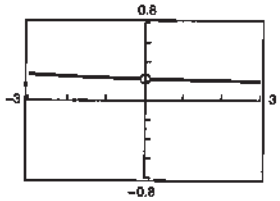


9.

x	-0.1	-0.01	-0.001	0
$f(x)$	0.2247	0.2237	0.2236	Error

x	0.001	0.01	0.1
$f(x)$	0.2236	0.2235	0.2225

0.2236;

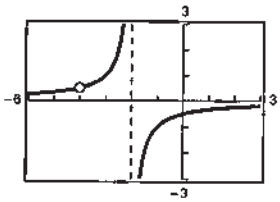


11.

x	-4.1	-4.01	-4.001	-4
$f(x)$	0.4762	0.4975	0.4998	Error

x	-3.999	-3.99	-3.9
$f(x)$	0.5003	0.5025	0.5263

$\frac{1}{2}$;

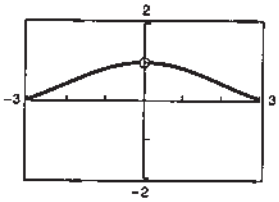


13.

x	-0.1	-0.01	-0.001	0
$f(x)$	0.9983	0.9999	0.9999	Error

x	0.001	0.01	0.1
$f(x)$	0.9999	0.9999	0.9983

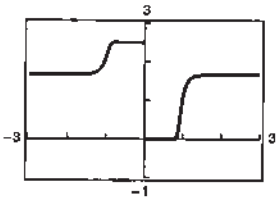
1;



15. 1 **17.** -1 **19.** Does not exist

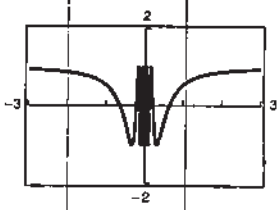
21. Does not exist

23.



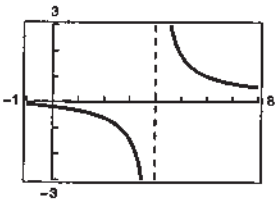
No

25.



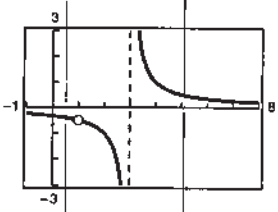
No

27.



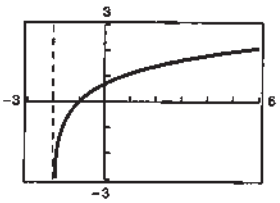
No

29.



Yes

31.



Yes

33. 0 **35.** 3 **37.** 5 **39.** 0 **41.** $e^3 \approx 20.08$

43. 0 **45.** $\frac{\pi}{6}$ **47.** Does not exist

49. (a) -12 (b) 9 (c) $\frac{1}{2}$ (d) $\sqrt{3}$

51. (a) 8 (b) $\frac{3}{8}$ (c) 3 (d) $-\frac{61}{8}$

53. True. This means that no matter how close x gets to c, there will be both positive and negative x-values that yield $f(x) = 3$ and $f(x) = -3$. This implies that the limit does not exist.

55. (a) Answers will vary. (b) Answers will vary.

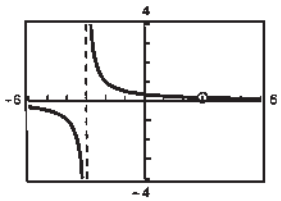
57. No. The function may approach different values from the right and left of 4. For example,

$$f(x) = \begin{cases} 0, & x < 2 \\ 4, & x = 2 \\ 6, & x > 2 \end{cases}$$

has $f(2) = 4$, but $\lim_{x \to 2} f(x) \neq 4$.

59. As a function's x-value approaches 5 from both the right and left sides, its corresponding output values approach 12.

61.



$\frac{1}{6}$; $x \neq \pm 3$; It may not be clear from a graph if a function is not defined at a single point. Examining a function graphically and algebraically ensures that you will find all points where the function is not defined.

63. $-\frac{1}{3}$, $x \neq 5$ **65.** $\frac{5x+4}{5x+2}$, $x \neq \frac{1}{3}$

67. $\frac{x^2 - 3x + 9}{x - 2}$, $x \neq -3$

69. 1 **71.** $\sqrt{70}$ **73.** $7\sqrt{2}$

Section 12.2 *(page 824)*

1. -15 **3.** $-\frac{9}{10}$ **5.** $\frac{7}{13}$ **7.** 1

9. (a) 1 (b) 3 (c) 5

$g_2(x) = -2x + 1$

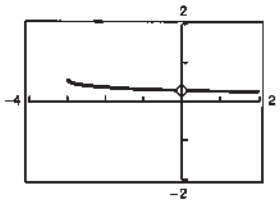
11. (a) 2 (b) 0 (c) 0

$g_2(x) = x(x + 1)$

13. $\frac{1}{12}$ **15.** 4 **17.** $\frac{1}{2\sqrt{5}}$ **19.** $\frac{1}{4}$

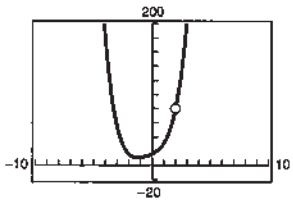
21. -1 **23.** Does not exist

25.



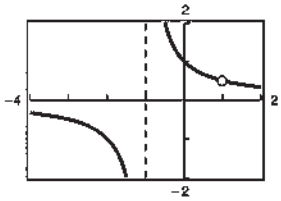
$\lim_{x \to 0} \frac{\sqrt{x+3} - \sqrt{3}}{x} \approx 0.2887$

27.



$\lim_{x \to 2} \frac{x^5 - 32}{x - 2} = 80$

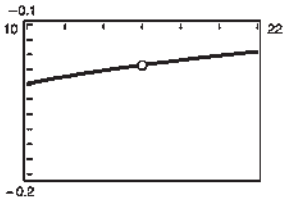
29.



x	0.9	0.99	0.999	0.9999
$f(x)$	0.5263	0.5025	0.5003	0.5000

$\lim_{x \to 1^-} \frac{(x-1)}{(x^2-1)} = 0.5$

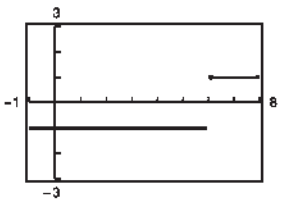
31.



x	16.1	16.01	16.001	16.0001
$f(x)$	-0.12481	-0.12498	-0.124998	-0.1249998

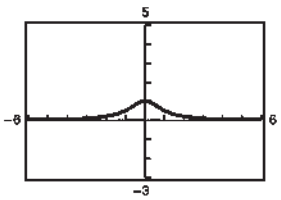
$\lim_{x \to 16^+} \frac{4 - \sqrt{x}}{x - 16} = -0.125$

33.



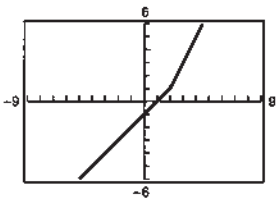
The limit does not exist.

35.



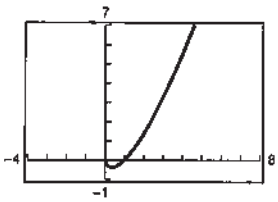
$\lim_{x \to 1} \frac{1}{x^2 + 1} = \frac{1}{2}$

37.



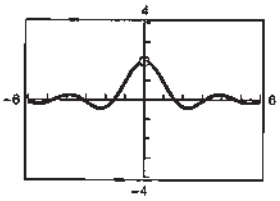
$\lim_{x \to 2} f(x)$ where $f(x) = \begin{cases} x - 1, & x \leq 2 \\ 2x - 3, & x > 2 \end{cases} = 1$

39.



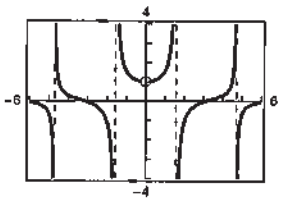
$\lim_{x \to 0^+} (x \ln x) = 0$

41.



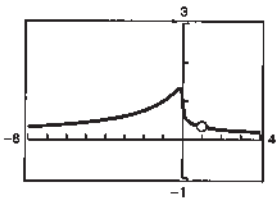
$\lim_{x \to 0} \frac{\sin 2x}{x} = 2$

43.



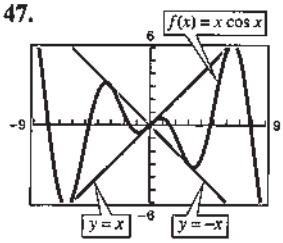
$\lim_{x \to 0} \frac{\tan x}{x} = 1$

45.



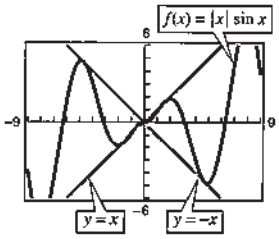
$\lim_{x \to 1} \frac{1 - \sqrt[3]{x}}{1 - x} = 0.333$

47.



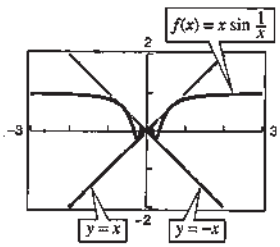
$\lim_{x \to 0} x \cos x = 0$

49.



$\lim_{x \to 0} |x| \sin x = 0$

51.



$\lim_{x \to 0} x \sin \frac{1}{x} = 0$

53. (a) 0. Direct substitution (b) 1

55. 3 **57.** $\frac{1}{2\sqrt{x}}$ **59.** $2x - 3$

61. -32 feet per second **63.** Answers will vary.

65. True **67. and 69.** Answers will vary.

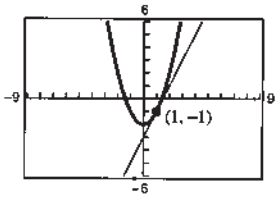
71. $x - 2y - 3 = 0$ **73.** Neither orthogonal nor parallel

75. Neither orthogonal nor parallel

Section 12.3 *(page 833)*

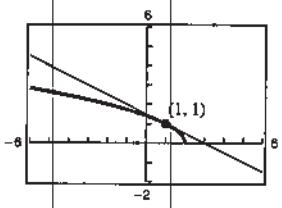
1. 0 **3.** $\frac{1}{2}$

5.



2

7.



$-\frac{1}{2}$

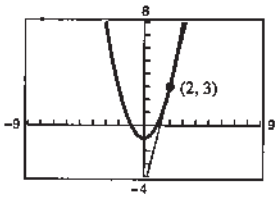
9. -2 **11.** 2 **13.** -1 **15.** $\frac{1}{6}$

17. $-2x$; (a) 0 (b) 2

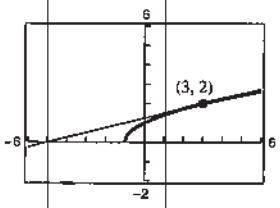
19. $-\frac{1}{(x+4)^2}$; (a) $-\frac{1}{16}$ (b) $-\frac{1}{4}$ **21.** 0

23. $-\frac{1}{3}$ **25.** $-\frac{2}{x^3}$

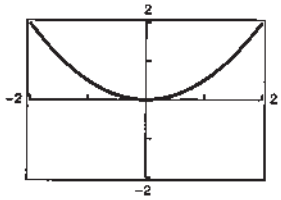
27. 4; $y = 4x - 5$



29. $\frac{1}{4}$; $y = \frac{1}{4}x + \frac{5}{4}$



31.

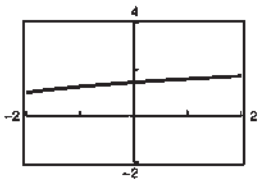


x	-2	-1.5	-1	-0.5	0
$f(x)$	2	1.125	0.5	0.125	0
$f'(x)$	-2	-1.5	-1	-0.5	0

x	0.5	1	1.5	2
$f(x)$	0.125	0.5	1.125	2
$f'(x)$	0.5	1	1.5	2

They appear to be the same.

33.



x	-2	-1.5	-1	-0.5	0
$f(x)$	1	1.225	1.414	1.581	1.732
$f'(x)$	0.5	0.408	0.354	0.316	0.289

x	0.5	1	1.5	2
$f(x)$	1.871	2	2.121	2.236
$f'(x)$	0.267	0.25	0.236	0.224

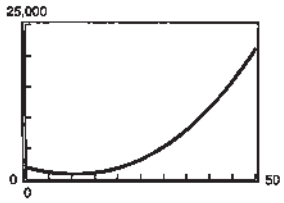
They appear to be the same.

35. $f'(x) = 2x - 4$; Horizontal tangent at $(2, -1)$

37. $f'(x) = 9x^2 - 9$; Horizontal tangents at $(-1, 6)$ and $(1, -6)$

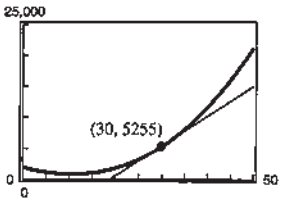
39. (a) $y = 0.073t^3 + 7.89t^2 - 192.4t + 1955$

(b)



500. In 1980, the per capita debt is increasing at the rate of \$500 per year.

(c)



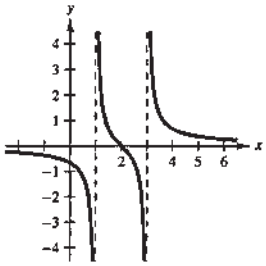
The slope given by the graphing utility (478.1) is close to the estimate (500).

41. True **43.** (b) **45.** (d)

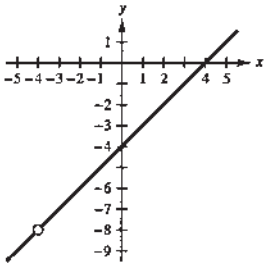
47. Answers will vary. Example: a sketch of any linear function with positive slope

49. Answers will vary. Example: a sketch of any quadratic function of the form $y = a(x - 1)^2 + k$

51.



53.



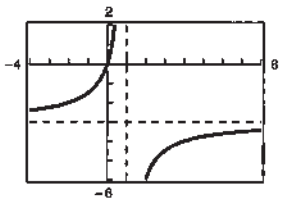
55. $\langle 0, 42, 0 \rangle$ **57.** $\langle -140, -46, 57 \rangle$

Section 12.4 *(page 841)*

1. (c) **3.** (d) **5.** 0 **7.** -1 **9.** 2

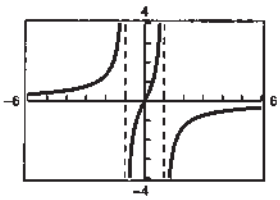
11. Does not exist **13.** -1 **15.** -4 **17.** -5

19.



Horizontal asymptote: $y = -3$

21.

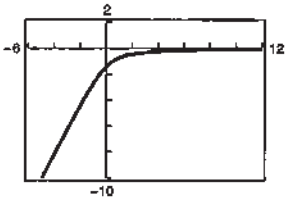


Horizontal asymptote: $y = 0$

23.

x	10^0	10^1	10^2	10^3
$f(x)$	-0.7321	-0.0995	-0.0100	-0.0010

x	10^4	10^5	10^6
$f(x)$	-1.0×10^{-4}	-1.0×10^{-5}	-1.0×10^{-6}

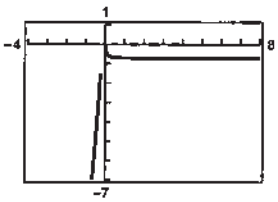


$\lim_{x \to \infty} f(x) = 0$

25.

x	10^0	10^1	10^2	10^3
$f(x)$	-0.7082	-0.7454	-0.7495	-0.74995

x	10^4	10^5	10^6
$f(x)$	-0.749995	-0.7499995	-0.75



$\lim_{x\to\infty} f(x) = -\frac{3}{4}$

27. $1, \frac{3}{5}, \frac{2}{5}, \frac{5}{17}, \frac{3}{13}$

Limit: 0

29. $\frac{1}{5}, \frac{1}{2}, \frac{9}{11}, \frac{8}{7}, \frac{25}{17}$

Limit does not exist.

31. 2, 3, 4, 5, 6

Limit does not exist.

33. $-1, \frac{1}{2}, -\frac{1}{3}, \frac{1}{4}, -\frac{1}{5}$

Limit: 0

35.

n	10^0	10^1	10^2	10^3
a_n	2	1.55	1.505	1.5005

n	10^4	10^5	10^6
a_n	1.50005	1.500005	1.5000005

$\lim_{n\to\infty} a_n = \frac{3}{2}$

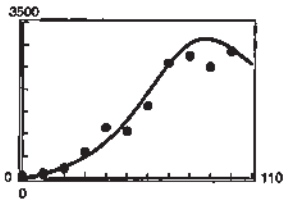
37.

n	10^0	10^1	10^2	10^3
a_n	16	6.16	5.4136	5.3413

n	10^4	10^5	10^6
a_n	5.3341	5.33341	5.333341

$\lim_{n\to\infty} a_n = \frac{16}{3}$

39. (a)



(b) 3022.9 thousand students

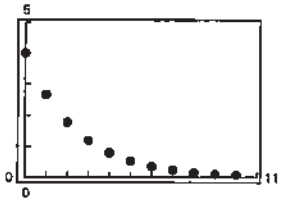
(c) As time approaches infinity, the number of high school graduates approaches zero.

41. False. Graph $y = \dfrac{x^2}{x+1}$.

43. Let $f(x) = \dfrac{1}{x^2}$, $g(x) = \dfrac{1}{x^2}$, and $c = 0$. Now

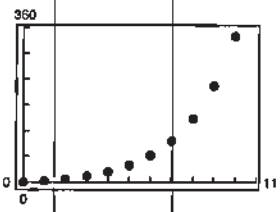
$$\lim_{x\to 0} \frac{1}{x^2} = \infty, \text{ and } \lim_{x\to 0}[f(x) - g(x)] = 0.$$

45.



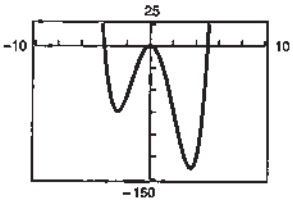
Converges to 0

47.

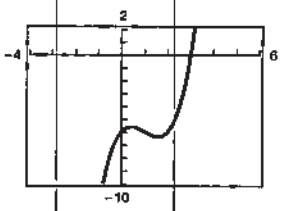


Diverges

49. $-4, 5, 0, 0$



51. 3



53. 60 **55.** 150

Section 12.5 *(page 849)*

1. 1830 **3.** 44,100 **5.** 5850

7. $S(n) = \dfrac{n^2 + 2n + 1}{4n^2}$

n	10^0	10^1	10^2	10^3
$S(n)$	1	0.3025	0.255025	0.25050025

Limit: $\frac{1}{4}$

9. $S(n) = \dfrac{2n^2 + 3n + 7}{2n^2}$

n	10^0	10^1	10^2	10^3
$S(n)$	6	1.185	1.0154	1.0015

Limit: 1

11. $S(n) = \dfrac{14n^2 + 3n + 1}{6n^3}$

n	10^0	10^1	10^2	10^3
$S(n)$	3	0.2385	0.02338	0.00233

Limit: 0

13. $S(n) = \dfrac{4n^2 - 3n - 1}{6n^2}$

n	10^0	10^1	10^2	10^3
$S(n)$	0	0.615	0.66165	0.66617

Limit: $\frac{2}{3}$

15. 14.25 **17.** 1.2656

19.

n	4	8	20	50
Approximate Area	18	21	22.8	23.52

21.

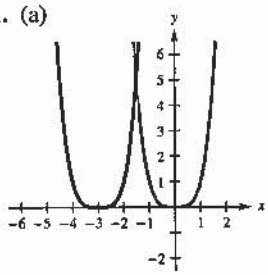
n	4	8	20	50
Approximate Area	3.52	2.85	2.48	2.34

23. 2 **25.** $\frac{10}{3}$ **27.** $\frac{17}{4}$ **29.** $\frac{51}{4}$ **31.** $\frac{1}{6}$ **33.** 1

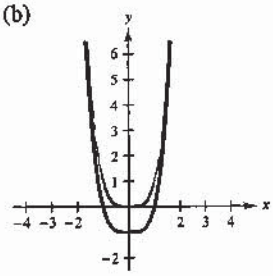
35. Area is 105,208.33 square feet $\approx$ 2.4153 acres

37. True **39.** Answers will vary.

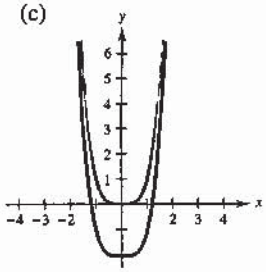
41. (a)



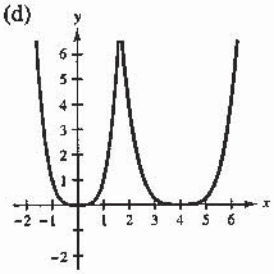
(b)



(c)



(d)



43. $\langle 24, -30 \rangle$ **45.** 18

Review Exercises *(page 852)*

1.

x	1.9	1.99	1.999	2
$f(x)$	0.1299	0.1255	0.1250	?

x	2.001	2.01	2.1
$f(x)$	0.1250	0.1245	0.1205

$$\lim_{x \to 2} \frac{x - 2}{3x^2 - 4x - 4} = \frac{1}{8}$$

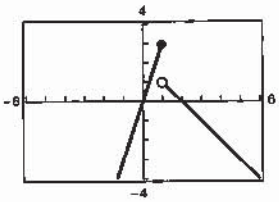
3. Yes **5.** No **7.** 5 **9.** $\frac{3}{10}$ **11.** 0

13. (a) 64 (b) 7 (c) 20 (d) $\frac{4}{5}$

15. 11 **17.** 77 **19.** $\frac{10}{3}$ **21.** $-\frac{1}{4}$ **23.** $\frac{1}{15}$

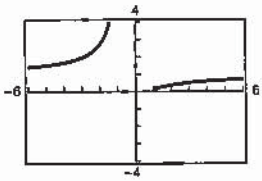
25. $-\frac{1}{3}$ **27.** -1 **29.** $\frac{1}{4}$ **31.** $\frac{1}{4}$

33.



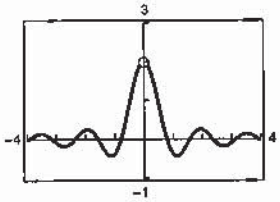
Does not exist

35.



Does not exist

37.



$\lim_{x \to 0} g(x) = 2$

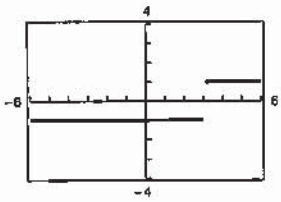
39.

x	1.1	1.01	1.001	1.0001
$f(x)$	0.5680	0.5764	0.5772	0.5773

$$\lim_{x \to 1^+} \frac{\sqrt{2x + 1} - \sqrt{3}}{x - 1} \approx 0.577. \left(\text{Actual limit is } \frac{\sqrt{3}}{3}\right).$$

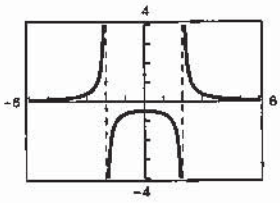
41. -1 **43.** 1

45.



Limit does not exist.

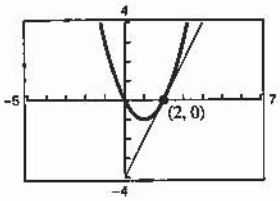
47.



Limit does not exist.

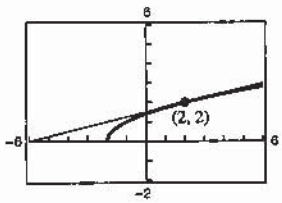
49. $3 - 2x$ **51.** Slope: 2

53.



2

55.



$\frac{1}{4}$

57. $2x - 4$; (a) -4 (b) 6

59. $-\dfrac{4}{(x-6)^2}$; (a) -4 (b) -1

61. $f'(x) = 0$ **63.** $g'(x) = 0$ **65.** $h'(x) = -\frac{1}{2}$

67. $f'(t) = \dfrac{1}{2\sqrt{t+5}}$ **69.** $g'(s) = -\dfrac{4}{(s+5)^2}$

71. 2 **73.** 0 **75.** 4

77. (a) $C = 22.50(100) + 12{,}200 = \$144.50$

$C = 22.50(1000) + 12{,}200 = \$34.70$

(b) \$22.50. As the number of calculators gets very large, the average cost approaches \$22.50.

79. $\frac{1}{2}, \frac{2}{5}, \frac{3}{10}, \frac{4}{17}, \frac{5}{26}$ **81.** $-2, 1, 2, \frac{5}{2}, \frac{14}{5}$

Limit: 0 Limit: 4

83. $S(n) = -\dfrac{3(n+1)(n+9)}{4n^2}$

n	10^0	10^1	10^2	10^3
$S(n)$	-15	-1.5675	-0.8257	-0.7575

Limit: $-\frac{3}{4}$

85. $\frac{113}{32} \approx 3.5313$

87.

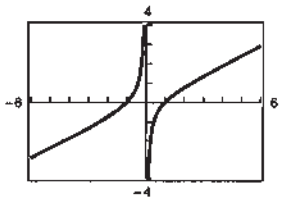
n	4	8	20	50
Approximate Area	10	10.5	10.64	10.6624

89. 9 **91.** $\frac{4}{3}$ **93.** 9

95. True. $\lim\limits_{x\to c}[f(x) + g(x)] = \lim\limits_{x\to c} f(x) + \lim\limits_{x\to c} g(x)$

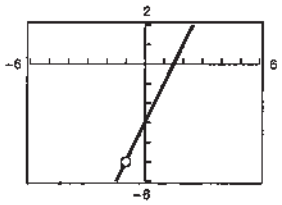
Chapter Test *(page 856)*

1.



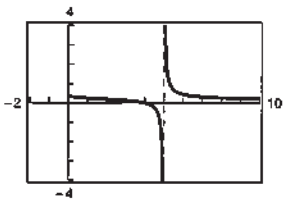
$\lim\limits_{x\to -2} \dfrac{x^2-1}{2x} = -\dfrac{3}{4}$

2.



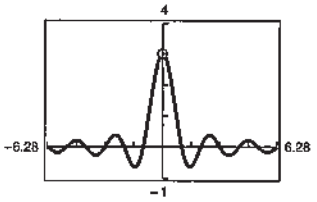
$\lim\limits_{x\to -1} \dfrac{2x^2-x-3}{x+1} = -5$

3.



The limit does not exist.

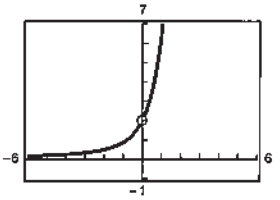
4.



$\lim\limits_{x\to 0} \dfrac{\sin 3x}{x} \approx 3$

x	-0.02	-0.01	0	0.01	0.02
$f(x)$	2.998	2.999	3	2.999	2.998

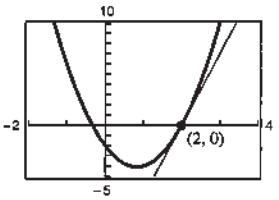
5.



$\lim\limits_{x\to 0} \dfrac{e^{2x}-1}{x} \approx 2$

x	-0.004	-0.003	-0.002	-0.001	0
$f(x)$	1.992	1.994	1.996	1.998	2

6.



$m = 7$

7. $6x^2 + 6$; 12 **8.** $-\frac{2}{3}$ **9.** $4x - 7$

10. $-\dfrac{1}{(x+7)^2}$ **11.** 0 **12.** -3

13. Does not exist **14.** $0, \frac{3}{4}, \frac{14}{19}, \frac{12}{17}, \frac{36}{53}$; Limit: $\frac{1}{2}$

15. $0, 1, 0, \frac{1}{2}, 0$; Limit: 0

16. $\frac{25}{2}$ **17.** 8 **18.** $\frac{3}{4}$

19. (a) $y = 8.79x^2 - 6.2x - 0.4$ (b) 81.7

Cumulative Test for Chapters 11–12
(page 857)

1. $(-4, 2, 3)$ **2.** $(0, -4, 0)$ **3.** $\sqrt{149}$

4. 3, 4, 5

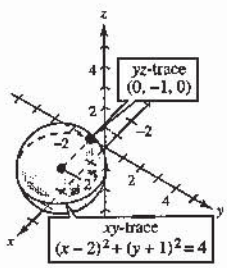
$3^2 + 4^2 \stackrel{?}{=} 5^2$

$9 + 16 \stackrel{?}{=} 25$

$25 = 25$

5. $\left(-1, 2, \frac{1}{2}\right)$ **6.** $(x-2)^2 + (y-2)^2 + (z-4)^2 = 24$

7.



8. -13; $8\mathbf{i} - 11\mathbf{j} - 20\mathbf{k}$ **9.** Neither

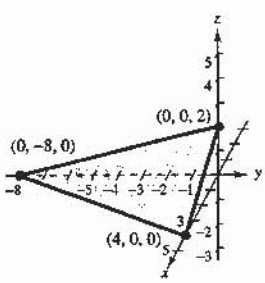
10. Orthogonal **11.** Parallel **12.** 12 cubic units

13. (a) $x = -2 + 7t,\ y = 3 + 5t,\ z = 25t$

(b) $\dfrac{x+2}{7} = \dfrac{y-3}{5} = \dfrac{z}{25}$

14. $75x + 50y - 31z = 0$

15.



16. $\dfrac{\sqrt{30}}{2} \approx 2.74$

17. 84.26° **18.** 4 **19.** $-\frac{1}{3}$ **20.** $\frac{1}{14}$ **21.** $\frac{1}{4}$

22. -1 **23.** 2 **24.** -2 **25.** $\frac{1}{2}$ **26.** $-\frac{1}{16}$

27. -4 **28.** Limit does not exist. **29.** $\frac{5}{2}$ **30.** 3

31. 0 **32.** 10,731.25 **33.** 8190 **34.** 672,880

35. $A \approx 8.125$ square units **36.** $A \approx 1.566$ square units

37. $\frac{3}{4}$ square unit **38.** 18 square units

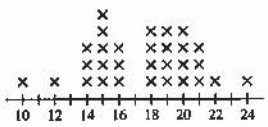
39. $\frac{2}{3}$ square unit

Appendix B

Section B.1 *(page A20)*

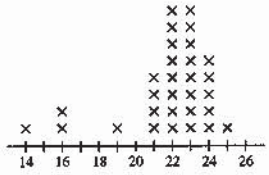
1. (a) \$1.109 (b) [\$0.999, \$1.189]

3. Quiz 1:



15

Quiz 2:



22 and 23

5.

Stems	*Leaves*
7	0 5 5 5 7 7 8 8 8
8	1 1 1 1 2 3 4 5 5 5 5 7 8 9 9 9
9	0 2 8
10	0 0

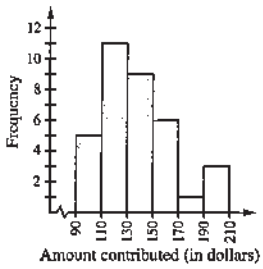
7.

Stems	*Leaves*
0	89 66 67 65 80 98 62 93
1	09 01 46 24 96 90
2	92 55 40 61
3	68 35
4	12 96 80 38
5	81 18 50 70 66
6	44 00 34 01
7	61 66 00
8	11 57 41 90
9	
10	
11	60 59 33
12	92
13	19 17 37
27	22
31	32
46	80
65	14

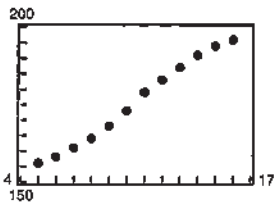
9. (a)

Interval	*Tally*
90–109	𝍸
110–129	𝍸 𝍸 |
130–149	𝍸 ||||
150–169	𝍸 |
170–189	|
190–209	|||

(b)



11. (a)

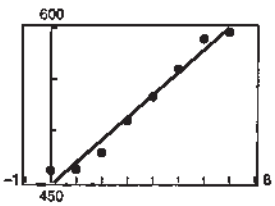


(b) Answers will vary. (c) $y = 4x + 134$

(d) 2000: 214; 2002: 222; 2005: 234

13. (a) $P = 21.91t + 447$

(b)



The model fits the data well.

(c)

Year	1990	1991	1992	1993
P (actual)	461.4	462.4	478.6	509.5
P (model)	447	468.9	490.8	512.7

Year	1994	1995	1996	1997
P (actual)	532.9	559.9	589.1	595.6
P (model)	534.6	556.6	578.5	600.4

Section B.2 *(page A28)*

1. Mean: 8.86; median: 8; mode: 7

3. Mean: 10.29; median: 8; mode: 7

5. Mean: 9; median: 8; mode: 7

7. The mean is sensitive to extreme values.

9. Mean: \$67.14; median: \$65.35

11. Mean: 3.07; median: 3; mode: 3

13. One possibility: $\{4, 4, 10\}$

15. The median gives the most representative description.

17. $\bar{x} = 6,\ v = 10,\ \sigma = 3.16$

19. $\bar{x} = 2,\ v = \frac{4}{3},\ \sigma = 1.15$ **21.** $\bar{x} = 4,\ v = 4,\ \sigma = 2$

23. $\bar{x} = 47,\ v = 226,\ \sigma = 15.03$ **25.** 3.42

27. 101.55 **29.** 1.65

31. (a) $\bar{x} = 12;\ \sigma = 2.83$ (b) $\bar{x} = 20;\ \sigma = 2.83$

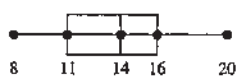
(c) $\bar{x} = 12;\ \sigma = 1.41$ (d) $\bar{x} = 9;\ \sigma = 1.41$

33. $\bar{x} = 12$ and $|x_i - 12| = 8$ for all x_i

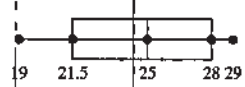
35. It will increase the mean by 5, but the standard deviation will not change.

37. First histogram

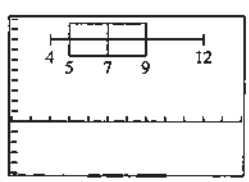
39.



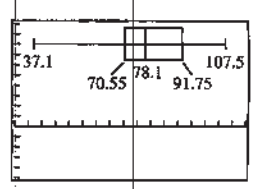
41.



43.



45.



Appendix C *(page A35)*

1. 3 **3.** 7 **5.** 4 **7.** 20 **9.** 4 **11.** 3

13. $\frac{4}{3}$ **15.** 20 **17.** No solution **19.** $x < 2$

21. $x < 9$ **23.** $x \le -14$ **25.** $x > 10$

27. $x < 4$ **29.** $x < 3$ **31.** $x \ge 2$ **33.** $x \le -5$

35. $x < 6$

Index of Applications

Biology and Life Sciences

Business

Chemistry and Physics

Construction

Consumer

Geometry

Interest Rate

Miscellaneous

Time and Distance

U.S. Demographics

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D

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N

O

P

T

U

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W

X

Y

Z

COMMON FORMULAS

Temperature

$F = \frac{9}{5}C + 32$

F = degrees Fahrenheit
C = degrees Celsius

Distance

$d = rt$

d = distance traveled
t = time
r = rate

Simple Interest

$I = Prt$

I = interest
P = principal
r = annual interest rate
t = time in years

Compound Interest

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

A = balance
P = principal
r = annual interest rate
n = compoundings per year
t = time in years

Coordinate Plane: Midpoint Formula

$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

midpoint of line segment joining (x_1, y_1) and (x_2, y_2)

Coordinate Plane: Distance Formula

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

d = distance between points (x_1, y_1) and (x_2, y_2)

Quadratic Formula

If $p(x) = ax^2 + bx + c$, $a \neq 0$ and $b^2 - 4ac \geq 0$, then the real zeros of p are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

CONVERSIONS

Length and Area

1 foot = 12 inches
1 mile = 5280 feet
1 kilometer = 1000 meters
1 kilometer ≈ 0.621 mile
1 meter ≈ 3.281 feet
1 foot ≈ 0.305 meter

1 yard = 3 feet
1 mile = 1760 yards
1 meter = 100 centimeters
1 mile ≈ 1.609 kilometers
1 meter ≈ 39.370 inches
1 foot ≈ 30.480 centimeters

1 meter = 1000 millimeters
1 centimeter ≈ 0.394 inch
1 inch ≈ 2.540 centimeters
1 acre = 4840 square yards
1 square mile = 640 acres

Volume

1 gallon = 4 quarts
1 gallon = 231 cubic inches
1 liter = 1000 milliliters
1 liter ≈ 1.057 quarts
1 gallon ≈ 3.785 liters

1 quart = 2 pints
1 gallon ≈ 0.134 cubic foot
1 liter = 100 centiliters
1 liter ≈ 0.264 gallon
1 quart ≈ 0.946 liter

1 pint = 16 fluid ounces
1 cubic foot ≈ 7.48 gallons

Weight and Mass on Earth

1 ton = 2000 pounds
1 kilogram ≈ 2.205 pounds

1 pound = 16 ounces
1 pound ≈ 0.454 kilogram

1 kilogram = 1000 grams
1 gram ≈ 0.035 ounce

FORMULAS FROM GEOMETRY

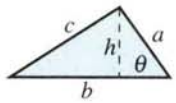
Triangle

$h = a \sin \theta$

Area $= \frac{1}{2}bh$

Laws of Cosines:

$c^2 = a^2 + b^2 - 2ab \cos \theta$



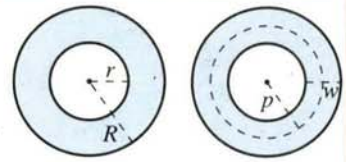
Circular Ring

Area $= \pi(R^2 - r^2)$

$= 2\pi pw$

(p = average radius,

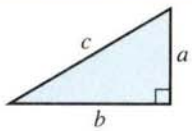
w = width of ring)



Right Triangle

Pythagorean Theorem:

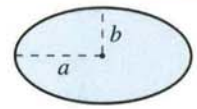
$c^2 = a^2 + b^2$



Ellipse

Area $= \pi ab$

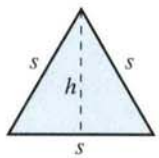
Circumference $\approx 2\pi\sqrt{\frac{a^2 + b^2}{2}}$



Equilateral Triangle

$h = \frac{\sqrt{3}s}{2}$

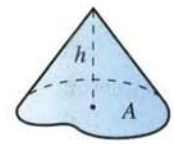
Area $= \frac{\sqrt{3}s^2}{4}$



Cone

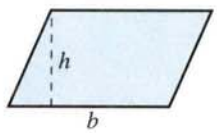
(A = area of base)

Volume $= \frac{Ah}{3}$



Parallelogram

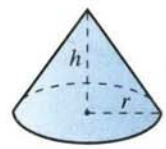
Area $= bh$



Right Circular Cone

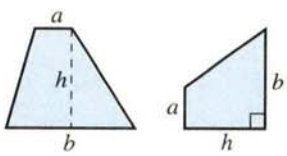
Volume $= \frac{\pi r^2 h}{3}$

Lateral Surface Area $= \pi r\sqrt{r^2 + h^2}$



Trapezoid

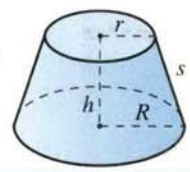
Area $= \frac{h}{2}(a + b)$



Frustum of Right Circular Cone

Volume $= \frac{\pi(r^2 + rR + R^2)h}{3}$

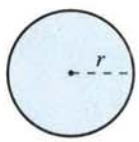
Lateral Surface Area $= \pi s(R + r)$



Circle

Area $= \pi r^2$

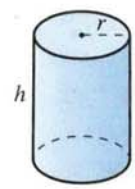
Circumference $= 2\pi r$



Right Circular Cylinder

Volume $= \pi r^2 h$

Lateral Surface Area $= 2\pi rh$

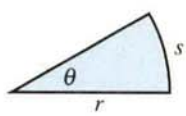


Sector of Circle

Area $= \frac{\theta r^2}{2}$

$s = r\theta$

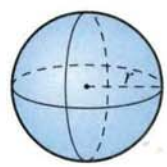
(θ in radians)



Sphere

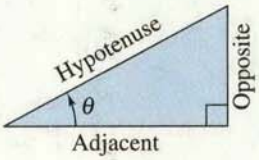
Volume $= \frac{4}{3}\pi r^3$

Surface Area $= 4\pi r^2$



Definition of the Six Trigonometric Functions

Right triangle definitions, where $0 < \theta < \pi/2$.

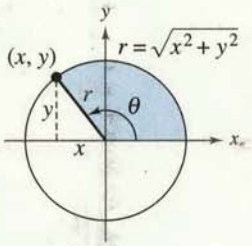


$$\sin\theta = \frac{\text{opp.}}{\text{hyp.}} \qquad \csc\theta = \frac{\text{hyp.}}{\text{opp.}}$$

$$\cos\theta = \frac{\text{adj.}}{\text{hyp.}} \qquad \sec\theta = \frac{\text{hyp.}}{\text{adj.}}$$

$$\tan\theta = \frac{\text{opp.}}{\text{adj.}} \qquad \cot\theta = \frac{\text{adj.}}{\text{opp.}}$$

Circular function definitions, where θ is any angle.



$$\sin\theta = \frac{y}{r} \qquad \csc\theta = \frac{r}{y}$$

$$\cos\theta = \frac{x}{r} \qquad \sec\theta = \frac{r}{x}$$

$$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{x}{y}$$

Reciprocal Identities

$$\sin u = \frac{1}{\csc u} \qquad \cos u = \frac{1}{\sec u} \qquad \tan u = \frac{1}{\cot u}$$

$$\csc u = \frac{1}{\sin u} \qquad \sec u = \frac{1}{\cos u} \qquad \cot u = \frac{1}{\tan u}$$

Quotient Identities

$$\tan u = \frac{\sin u}{\cos u} \qquad \cot u = \frac{\cos u}{\sin u}$$

Pythagorean Identities

$$\sin^2 u + \cos^2 u = 1$$

$$1 + \tan^2 u = \sec^2 u \qquad 1 + \cot^2 u = \csc^2 u$$

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - u\right) = \cos u \qquad \cot\left(\frac{\pi}{2} - u\right) = \tan u$$

$$\cos\left(\frac{\pi}{2} - u\right) = \sin u \qquad \sec\left(\frac{\pi}{2} - u\right) = \csc u$$

$$\tan\left(\frac{\pi}{2} - u\right) = \cot u \qquad \csc\left(\frac{\pi}{2} - u\right) = \sec u$$

Even/Odd Identities

$$\sin(-u) = -\sin u \qquad \cot(-u) = -\cot u$$

$$\cos(-u) = \cos u \qquad \sec(-u) = \sec u$$

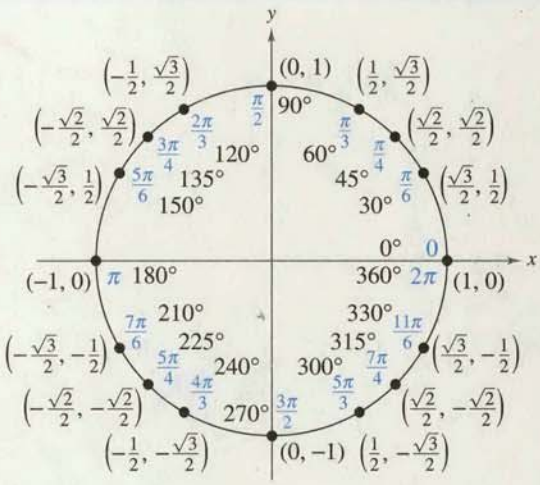
$$\tan(-u) = -\tan u \qquad \csc(-u) = -\csc u$$

Sum and Difference Formulas

$$\sin(u \pm v) = \sin u \cos v + \cos u \sin v$$

$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$

$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$$



Double-Angle Formulas

$$\sin 2u = 2 \sin u \cos u$$

$$\cos 2u = \cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$$

$$\tan 2u = \frac{2 \tan u}{1 - \tan^2 u}$$

Power-Reducing Formulas

$$\sin^2 u = \frac{1 - \cos 2u}{2}$$

$$\cos^2 u = \frac{1 + \cos 2u}{2}$$

$$\tan^2 u = \frac{1 - \cos 2u}{1 + \cos 2u}$$

Sum-to-Product Formulas

$$\sin u + \sin v = 2 \sin\left(\frac{u+v}{2}\right) \cos\left(\frac{u-v}{2}\right)$$

$$\sin u - \sin v = 2 \cos\left(\frac{u+v}{2}\right) \sin\left(\frac{u-v}{2}\right)$$

$$\cos u + \cos v = 2 \cos\left(\frac{u+v}{2}\right) \cos\left(\frac{u-v}{2}\right)$$

$$\cos u - \cos v = -2 \sin\left(\frac{u+v}{2}\right) \sin\left(\frac{u-v}{2}\right)$$

Product-to-Sum Formulas

$$\sin u \sin v = \frac{1}{2}[\cos(u - v) - \cos(u + v)]$$

$$\cos u \cos v = \frac{1}{2}[\cos(u - v) + \cos(u + v)]$$

$$\sin u \cos v = \frac{1}{2}[\sin(u + v) + \sin(u - v)]$$

$$\cos u \sin v = \frac{1}{2}[\cos(u + v) - \sin(u - v)]$$