

A quadratic equation:  $ax^2 + bx + c = 0$

Can be solved using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example: Solve the equation:  $3x^2 + 23x + 40 = 0$

$$a = 3$$

$$b = 23$$

$$c = 40$$

$$b^2 - 4ac$$

$$23^2 - 4(3)(40)$$

$$49$$

$$x = \frac{-23 \pm \sqrt{49}}{2(3)}$$

$$x = \frac{-23 \pm 7}{6}$$

$$x = \frac{-23+7}{6}$$

$$x = \frac{-23-7}{6}$$

$$x = \frac{-16}{6}$$

$$x = \frac{-30}{6}$$

$$x = \frac{-8}{3}$$

$$x = -5$$

Example: Solve the equation:  $3x^2 + 2x = 2$

$$3x^2 + 2x - 2 = 0$$

$$a = 3$$

$$b = 2$$

$$c = -2$$

$$b^2 - 4ac$$

$$(2)^2 - 4(3)(-2)$$

$$4 - (-24) = 28$$

$$x = \frac{-2 \pm \sqrt{28}}{2(3)}$$

$$x = \frac{-2 \pm \sqrt{4 \cdot 7}}{6}$$

$$x = \frac{-2 \pm 2\sqrt{7}}{6}$$

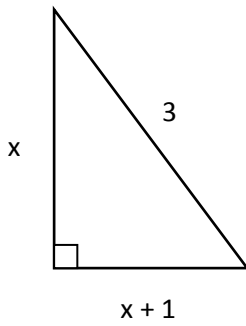
$$x = \frac{-2}{6} \pm \frac{2\sqrt{7}}{6}$$

$$x = \frac{-1}{3} \pm \frac{\sqrt{7}}{3}$$

Example: Solve the equation:  $5x^2 + 14x - 3 = 0$

Example: Solve the equation:  $x^2 - 3x = 9$

SAT Question: The longer leg of a right triangle is 1 cm longer than the shorter leg. The hypotenuse is 3 cm long. What is the length of the shortest side?



$x = \text{shorter leg}$

$x + 1 = \text{longer leg}$

$$c^2 = a^2 + b^2$$

$$3^2 = x^2 + (x + 1)^2$$

$$9 = x^2 + x^2 + 2x + 1$$

$$0 = 2x^2 + 2x - 8$$

$a = 2$
$b = 2$
$c = -8$

$$b^2 - 4ac$$

$$2^2 - 4(2)(-8)$$

$$4 + 64 = 68$$

$$x = \frac{-2 \pm \sqrt{68}}{2(2)}$$

$$x \approx \frac{-2 \pm 8.25}{4}$$

$$x \approx \frac{-2 + 8.25}{4} \quad x \approx \frac{-2 - 8.25}{4}$$

$$x \approx 1.56$$

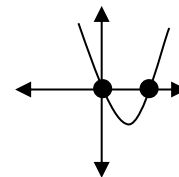
$$x \approx -2.56$$

length of side must be positive  
so  $x \approx 1.56$

## The Discriminant $b^2 - 4ac$

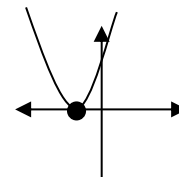
when  $b^2 - 4ac > 0$

2 real solutions



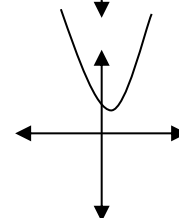
when  $b^2 - 4ac = 0$

1 real solution



when  $b^2 - 4ac < 0$

0 real solutions  
(2 imaginary solutions)



## Using the Discriminant Notes

Determine the type and number of solutions in each quadratic equation

Example:  $x^2 + 6x + 8 = 0$

$$\underline{b^2 - 4ac}$$

$$6^2 - 4(1)(8)$$

$$36 - 32 = 4$$

2 real solutions

Example:  $x^2 + 6x + 9 = 0$

$$\underline{b^2 - 4ac}$$

$$6^2 - 4(1)(9)$$

$$36 - 36 = 0$$

1 real solution

Example:  $x^2 + 6x + 10 = 0$

$$\underline{b^2 - 4ac}$$

$$6^2 - 4(1)(10)$$

$$36 - 40 = -4$$

0 real solutions (2 imaginary solutions)

Example:  $2x^2 + 7x - 18 = 0$

$$\underline{b^2 - 4ac}$$

Example:  $x^2 - x + 6 = 0$

$$\underline{b^2 - 4ac}$$

Example:  $4x^2 - 12x = -9$

$$\underline{b^2 - 4ac}$$