Convergence and Divergence Recap

Will the Series converge or diverge?

Try the ***n*-th term test *first***. It can be used on any a_n but only tests for divergence.

1. $\sum_{n=0}^{\infty}(-1)^n(1)$

- ***n*-th term** test: $a_{n=}(-1)^n(1)$ and $\lim_{n\to\infty}(-1)^n(1) \neq 0$, therefore the series diverges.
- **NOTE:** This is an alternating series BUT for the **Alternating Series Test (AST)** your $a_n = 1$. You disregard the alternator. Since your term are always equal to 1, the terms do NOT decrease. This test cannot be used.

2. $\sum_{n=1}^{\infty}(-1)^{n-1}(n)$

- ***n*-th term test**: $a_{n=}(-1)^{n-1}(n)$ and $\lim_{n\to\infty}(-1)^{n-1}(n) \neq 0$, therefore the series diverges.
- **NOTE:** This is an alternating series BUT for the **AST** your $a_n = n$ which again does NOT decrease, and this test cannot be used.

3. $\sum_{n=0}^{\infty}\frac{(-1)1^{n+1}}{n+1} = \sum_{n=0}^{\infty}\frac{-1}{n+1} = -\sum_{n=0}^{\infty}\frac{1}{n+1}$

- **NOTE:** 1^{n+1} will always equal 1 and "factoring out" <u>any</u> constant like the -1 will ***<u>not</u>*** change whether a series converges or diverges. For example, if the positive series diverges to $+\infty$ then the negative series will still diverge but to $-\infty$.
- ***n*-th term test**: $a_{n=}\frac{1}{n+1}$ and $\lim_{n\to\infty}\frac{1}{n+1} = 0$. Test is inconclusive.
- Not a **Geometric Series**
- We cannot use **AST** because series does NOT alternate.
- **Ratio Test:** $\lim_{n\to\infty}\left|\frac{-1}{(n+1)+1}\cdot\frac{n+1}{-1}\right| = \lim_{n\to\infty}\left|\frac{n+1}{n+2}\right| = 1$. Test cannot be used.

Any of the following tests can be used for #3. They produce the same answer.

- **Integral Test:** $f(x) = \frac{1}{x+1}$ is continuous over the interval, positive and decreasing. $\int_1^{\infty}\frac{1}{x+1}dx = \lim_{b\to\infty}\int_1^{b}\frac{1}{x+1}dx = \infty$. The series diverges. You might want to practice evaluating Improper Integrals to verify.
- ***p*-series Test/Direct Comparison:** The Harmonic Series $\frac{1}{n}$ diverges, so the *n* in the denominator of $\frac{1}{n+1}$ will dominate the limit and the $+1$ will have no effect so $\frac{1}{n+1}$ will also diverge.
- **Limit Comparison:** $a_n = \frac{1}{n+1}, b_n = \frac{1}{n}$; $L = \lim_{n\to\infty}\frac{a_n}{b_n} = \lim_{n\to\infty}\left(\frac{1}{n+1}\cdot\frac{n}{1}\right) = 1$. Since $L > 0$ *and* $\sum_{n=1}^{\infty}\frac{1}{n}$ diverges so does $\sum_{n=1}^{\infty}\frac{1}{n+1}$.

Absolute convergence implies convergence. What does that mean?

4. $\sum_{n=1}^{\infty}\frac{(-1)^{n+2}}{n^2}$

- Let's check absolute convergence $\sum_{n=1}^{\infty}\left|\frac{(-1)^{n+2}}{n^2}\right| = \sum_{n=1}^{\infty}\frac{1}{n^2}$
- This is a ***p*-series** with $p > 1$. Since the series has absolute convergence, we get convergence for "free" and do not need to use any other test.
- **NOTE:** This is only true for convergence