CHA			
1	Topic: 4.4	Introduction to Related Rates	
3	Topic: 4.5	Solving Related Rates Problems	
Learning Objective CHA-3.D: Calculate related rates in applied contexts.			
Learning Objective CHA-3.E: Interpret related rates in applied contexts.			

Our next application of the derivative will focus on two (or more) rates that are simultaneously changing. We call these types of problems "related rates problems."

Example 1: Write the following statements mathematically.

a. John is growing at the rate of 3 inches/year	**b.** My mutual fund is shrinking by 4 cents/day
c. The radius of a circle is increasing by 4 ft/hr	**d.** The volume of a cone is decreasing by 2 in^3/sec

To solve related rates problems, you need a strategy that always works. Related rates problems always can be recognized by the words "**increasing, decreasing, growing, shrinking, changing**." Follow these guidelines in solving a related rates problem.

Step 1: Make a sketch. Label all sides in terms of variables even if you are given the actual values of the sides.
Step 2: Write down the sides and the rates that are given. Write down the rate you are trying to find.
Step 3: Find an equation that ties your variables together.
If the problem mentions area, you will most likely write an area formula.
If the problem mentions volume, you will most likely write a volume formula.
If the problem mentions lengths of sides and the figure is a right triangle, you will most likely use the Pythagorean Theorem as your equation.
If the problem mentions angles measures and the figure is a right triangle, you will most likely use a trigonometric ratio as your equation.
Step 4: You may now plug in any **constant** into your equation. **Never** plug in any variable into your equation until **after** the derivative is taken.
Step 5: Differentiate your equation with respect to time. Remember, you are implicitly differentiating with respect to *t*.
Step 6: Plug in all variable quantities mentioned in the "given" and "when" parts of the problem. Hopefully, you will know all the variables except for one. If not, you will need an equation which will solve for unknown variables. Sometimes this will require using the same equation as the one you used above. Do this work off to the side.
Step 7: Label your answers in terms of the correct units (very important) and be sure you answered the question asked.

Example 2: ***Rectangle***

A rectangle's length is increasing at the rate of 2 inches/sec and its width is increasing at rate of 3 inches/sec. Find how fast the perimeter is changing at the moment its length is 10 inches and its width is 6 inches.

Picture (only label variable information) *Given:* *Find:*

Equation:

Example 3: ***Right Triangle***

A right triangle has sides whose lengths are changing. The short side is increasing at 3 in./sec and the long side is decreasing at 5 in/sec. Find the rate of change of the area of the triangle at the moment the short side is 30 inches and the long side is 40 inches.

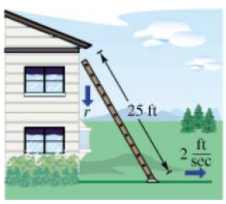
Given: *Find:*

Picture (only label variable information)

Equation:

Example 4: ***A Falling Ladder***

Justin is paying a late-night visit to his girlfriend, Jessica in hopes of sneaking her out of her house after curfew. He brings a 25-foot ladder and leans it against the side of her house to climb to her window. Soon after Justin reaches the top of the ladder, Jessica's father pulls into the driveway, rushes to the ladder and begins to pull its lower end away from the house at a rate 2 feet per second. How fast is Justin plummeting to the ground if he continues to hang onto the top of the ladder at the instant the bottom of the ladder is 7 feet from the house?



Example 5: ***Cylinder***

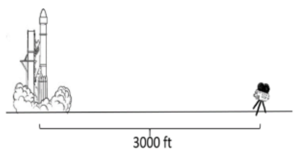
A right circular cylinder has a height and radius whose dimensions are each changing.
The radius is growing at 2 feet/min and the height is shrinking at 3 feet/min. Find the rate of change of the volume of the cylinder at the moment the height is 10 feet and the radius is 8 feet.

Picture (only label variable information) *Given:* *Find:*

Equation:

<u>Example 6</u>: ***A Rotating Camera***

A camera is mounted 3,000 feet from a rocket's launching pad. The camera needs to pivot as the rocket is launched and needs to keep the rocket in focus. If the rocket is rising vertically at 800 feet/sec, how fast is the angle of elevation of the camera changing at that moment when the shuttle is 4,000 feet high?



Example 7: ***A Pile of Sand (Cone Problem 1)***
Sand is poured on a beach creating a cone whose radius is always equal to twice its height. If the sand is poured at the rate of 20 in^3/sec, how fast is the height of the conical pile changing at the time the height is 2 inches?

Example 8: ***A Draining Tank of Water (Cone Problem 2)***
Water is draining from a conical tank (with vertex down) at the rate of 2 $meters^3$/sec. The tank is 16 meters high and its top radius is 4 meters. How fast is the water level falling when the water level is 12 meters high?