

Factoring a Sum or Difference of Cubes

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Example: Factor: $x^3 - 27$

Factor: $8x^3 - 1$

$$\begin{aligned} (x)^3 - (3)^3 &= (x - 3)(x^2 + (x)(3) + (3)^2) \\ &= (x - 3)(x^2 + 3x + 9) \end{aligned}$$

Example: Factor: $2x^4 + 16$

Example: Solve: $8x^3 + 125 = 0$

Find all complex roots.

$$(2x)^3 + (5)^3$$

$$(2x + 5)(4x^2 - 10x + 25) = 0$$

$$2x + 5 = 0$$

$$4x^2 - 10x + 25 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$2x = -5$$

$$a = 4$$

$$\frac{b^2 - 4ac}{2(4)}$$

$$x = \frac{+10 \pm \sqrt{-300}}{2(4)}$$

$$x = -\frac{5}{2}$$

$$b = -10$$

$$(-10)^2 - 4(4)(25)$$

$$x = \frac{10 \pm i\sqrt{300}}{8}$$

$$c = 25$$

$$= -300$$

$$x = \frac{10 \pm 10i\sqrt{3}}{8}$$

$$x = \frac{10}{8} \pm \frac{10i\sqrt{3}}{8}$$

$$x = \frac{5}{4} \pm \frac{5i\sqrt{3}}{4}$$

Factoring and solving quartic trinomials.

Factor: $x^4 - 6x^2 - 27$

$$(x^2 - 9)(x^2 + 3)$$

$$(x + 3)(x - 3)(x^2 + 3)$$

Solve: $x^4 - 4x^2 - 45 = 0$