

AP CALCULUS
Stuff you **MUST** know Cold

* means topic only on BC

<p>Curve sketching and analysis $y = f(x)$ must be continuous at each: critical point: $\frac{dy}{dx} = 0$ or <u>undefined</u> local minimum: and look out for endpoints $\frac{dy}{dx}$ goes $(-,0,+)$ or $(-,und,+)$ or $\frac{d^2y}{dx^2} > 0$ local maximum: $\frac{dy}{dx}$ goes $(+,0,-)$ or $(+,und,-)$ or $\frac{d^2y}{dx^2} < 0$ point of inflection: concavity changes $\frac{d^2y}{dx^2}$ goes from $(+,0,-)$, $(-,0,+)$, $(+,und,-)$, or $(-,und,+)$</p>	<p align="center">Differentiation Rules</p> <p>Chain Rule $\frac{d}{dx}[f(u)] = f'(u) \frac{du}{dx}$ OR $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$</p> <p>Product Rule $\frac{d}{dx}(uv) = \frac{du}{dx}v + u \frac{dv}{dx}$ OR $u'v + uv'$</p> <p>Quotient Rule $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{\frac{du}{dx}v - u \frac{dv}{dx}}{v^2}$ OR $\frac{u'v - uv'}{v^2}$</p>	<p align="center">Approx. Methods for Integration</p> <p>Trapezoidal Rule $\int_a^b f(x)dx = \frac{1}{2} \frac{b-a}{n} [f(x_0) + 2f(x_1) + \dots + 2f(x_{n-1}) + f(x_n)]$</p> <p>Simpson's Rule $\int_a^b f(x)dx = \frac{1}{3} \Delta x [f(x_0) + 4f(x_1) + 2f(x_2) + \dots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]$</p>
<p align="center">Basic Derivatives</p> <p>$\frac{d}{dx}(x^n) = nx^{n-1}$ $\frac{d}{dx}(\sin x) = \cos x$ $\frac{d}{dx}(\cos x) = -\sin x$ $\frac{d}{dx}(\tan x) = \sec^2 x$ $\frac{d}{dx}(\cot x) = -\csc^2 x$ $\frac{d}{dx}(\sec x) = \sec x \tan x$ $\frac{d}{dx}(\csc x) = -\csc x \cot x$ $\frac{d}{dx}(\ln u) = \frac{1}{u} \frac{du}{dx}$ $\frac{d}{dx}(e^u) = e^u \frac{du}{dx}$ where u is a function of x, and a is a constant.</p>	<p align="center">“PLUS A CONSTANT”</p> <hr/> <p align="center">The Fundamental Theorem of Calculus</p> <p>$\int_a^b f(x)dx = F(b) - F(a)$ where $F'(x) = f(x)$</p>	<p align="center">Theorem of the Mean Value i.e. AVERAGE VALUE</p> <p>If the function $f(x)$ is continuous on $[a, b]$ and the first derivative exists on the interval (a, b), then there exists a number $x = c$ on (a, b) such that</p> $f(c) = \frac{\int_a^b f(x)dx}{(b-a)}$ <p>This value $f(c)$ is the “average value” of the function on the interval $[a, b]$.</p>
<p align="center">More Derivatives</p> <p>$\frac{d}{dx}\left(\sin^{-1} \frac{u}{a}\right) = \frac{1}{\sqrt{a^2 - u^2}} \frac{du}{dx}$ $\frac{d}{dx}\left(\cos^{-1} x\right) = \frac{-1}{\sqrt{1 - x^2}}$ $\frac{d}{dx}\left(\tan^{-1} \frac{u}{a}\right) = \frac{a}{a^2 + u^2} \cdot \frac{du}{dx}$ $\frac{d}{dx}\left(\cot^{-1} x\right) = \frac{-1}{1 + x^2}$ $\frac{d}{dx}\left(\sec^{-1} \frac{u}{a}\right) = \frac{a}{ u \sqrt{u^2 - a^2}} \cdot \frac{du}{dx}$ $\frac{d}{dx}(\csc^{-1} x) = \frac{-1}{ x \sqrt{x^2 - 1}}$ $\frac{d}{dx}(a^{u(x)}) = a^{u(x)} \ln a \cdot \frac{du}{dx}$ $\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$</p>	<p align="center">Mean Value Theorem</p> <p>If the function $f(x)$ is continuous on $[a, b]$, AND the first derivative exists on the interval (a, b), then there is at least one number $x = c$ in (a, b) such that</p> $f'(c) = \frac{f(b) - f(a)}{b - a}$	<p align="center">Distance, Velocity, and Acceleration</p> <p>velocity = $\frac{d}{dt}$ (position) acceleration = $\frac{d}{dt}$ (velocity) *velocity vector = $\left\langle \frac{dx}{dt}, \frac{dy}{dt} \right\rangle$ speed = $v = \sqrt{(x')^2 + (y')^2}$ * displacement = $\int_{t_0}^{t_f} v dt$ distance = $\int_{\text{initial time}}^{\text{final time}} v dt$ $\int_{t_0}^{t_f} \sqrt{(x')^2 + (y')^2} dt$ * average velocity = $= \frac{\text{final position} - \text{initial position}}{\text{total time}}$ $= \frac{\Delta x}{\Delta t}$</p>
	<p align="center">Rolle's Theorem</p> <p>If the function $f(x)$ is continuous on $[a, b]$, AND the first derivative exists on the interval (a, b), AND $f(a) = f(b)$, then there is at least one number $x = c$ in (a, b) such that</p> $f'(c) = 0$	

BC TOPICS and important TRIG identities and values

<p style="text-align: center;">L'Hôpital's Rule</p> <p>If $\frac{f(a)}{g(b)} = \frac{0}{0}$ or $\frac{\infty}{\infty}$,</p> <p>then $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$</p>	<p style="text-align: center;">Slope of a Parametric equation</p> <p>Given a $x(t)$ and a $y(t)$ the slope is</p> $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$	<p style="text-align: center;">Values of Trigonometric Functions for Common Angles</p> <table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <thead> <tr> <th>θ</th> <th>$\sin \theta$</th> <th>$\cos \theta$</th> <th>$\tan \theta$</th> </tr> </thead> <tbody> <tr> <td>0°</td> <td>0</td> <td>1</td> <td>0</td> </tr> <tr> <td>$\frac{\pi}{6}$</td> <td>$\frac{1}{2}$</td> <td>$\frac{\sqrt{3}}{2}$</td> <td>$\frac{\sqrt{3}}{3}$</td> </tr> <tr> <td>$\frac{\pi}{4}$</td> <td>$\frac{\sqrt{2}}{2}$</td> <td>$\frac{\sqrt{2}}{2}$</td> <td>1</td> </tr> <tr> <td>$\frac{\pi}{3}$</td> <td>$\frac{\sqrt{3}}{2}$</td> <td>$\frac{1}{2}$</td> <td>$\sqrt{3}$</td> </tr> <tr> <td>$\frac{\pi}{2}$</td> <td>1</td> <td>0</td> <td>"∞"</td> </tr> <tr> <td>π</td> <td>0</td> <td>-1</td> <td>0</td> </tr> </tbody> </table> <p style="font-size: small; color: blue;">Know both the <i>inverse trig</i> and the <i>trig</i> values. E.g. $\tan(\pi/4)=1$ & $\tan^{-1}(1)=\pi/4$</p>	θ	$\sin \theta$	$\cos \theta$	$\tan \theta$	0°	0	1	0	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\pi}{2}$	1	0	" ∞ "	π	0	-1	0
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π	0	-1	0																											
<p style="text-align: center;">Euler's Method</p> <p>If given that $\frac{dy}{dx} = f(x, y)$ and that the solution passes through (x_0, y_0),</p> <p>$y(x_0) = y_0$</p> <p style="text-align: center;">\vdots</p> <p>$y(x_n) = y(x_{n-1}) + f(x_{n-1}, y_{n-1}) \cdot \Delta x$</p> <p>In other words:</p> $x_{\text{new}} = x_{\text{old}} + \Delta x$ $y_{\text{new}} = y_{\text{old}} + \left. \frac{dy}{dx} \right _{(x_{\text{old}}, y_{\text{old}})} \cdot \Delta x$	<p style="text-align: center;">Polar Curve</p> <p>For a polar curve $r(\theta)$, the AREA inside a "leaf" is</p> $\int_{\theta_1}^{\theta_2} \frac{1}{2} [r(\theta)]^2 d\theta$ <p>where θ_1 and θ_2 are the "first" two times that $r = 0$.</p> <p>The SLOPE of $r(\theta)$ at a given θ is</p> $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\frac{d}{d\theta} [r(\theta) \sin \theta]}{\frac{d}{d\theta} [r(\theta) \cos \theta]}$	<p style="text-align: center;">Trig Identities</p> <p><i>Double Argument</i></p> <p>$\sin 2x = 2 \sin x \cos x$</p> <p>$\cos 2x = \cos^2 x - \sin^2 x = 1 - 2 \sin^2 x$</p> <p>$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$</p> <p>$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$</p> <p><i>Pythagorean</i></p> <p>$\sin^2 x + \cos^2 x = 1$ (others are easily derivable by dividing by $\sin^2 x$ or $\cos^2 x$)</p> <p>$1 + \tan^2 x = \sec^2 x$</p> <p>$\cot^2 x + 1 = \csc^2 x$</p> <p><i>Reciprocal</i></p> <p>$\sec x = \frac{1}{\cos x}$ or $\cos x \sec x = 1$</p> <p>$\csc x = \frac{1}{\sin x}$ or $\sin x \csc x = 1$</p> <p><i>Odd-Even</i></p> <p>$\sin(-x) = -\sin x$ (odd)</p> <p>$\cos(-x) = \cos x$ (even)</p> <p><u>Some more handy INTEGRALS:</u></p> <p>$\int \tan x dx = \ln \sec x + C$</p> <p style="text-align: center;">$= -\ln \cos x + C$</p> <p>$\int \sec x dx = \ln \sec x + \tan x + C$</p>																												
<p style="text-align: center;">Integration by Parts</p> $\int u dv = uv - \int v du$	<p style="text-align: center;">Ratio Test</p> <p>The series $\sum_{k=0}^{\infty} a_k$ converges if</p> $\lim_{k \rightarrow \infty} \left \frac{a_{k+1}}{a_k} \right < 1$ <p style="color: green;">If the limit equal 1, you know nothing.</p>	<p style="text-align: center;">Integral of Log</p> <p>Use IBP and let $u = \ln x$ (Recall $u = \text{LIPET}$ or LIATE)</p> $\int \ln x dx = x \ln x - x + C$																												
<p style="text-align: center;">Taylor Series</p> <p>If the function f is "smooth" at $x = a$, then it can be approximated by the n^{th} degree polynomial</p> <p>$f(x) \approx f(a) + f'(a)(x-a)$</p> $+ \frac{f''(a)}{2!} (x-a)^2 + \dots$ $+ \frac{f^{(n)}(a)}{n!} (x-a)^n.$	<p style="text-align: center;">Lagrange Error Bound</p> <p>If $P_n(x)$ is the n^{th} degree Taylor polynomial of $f(x)$ about c and $f^{(n+1)}(t) \leq M$ for all t between x and c, then</p> $ f(x) - P_n(x) \leq \frac{M}{(n+1)!} x-c ^{n+1}$	<p style="text-align: center;">Maclaurin Series</p> <p>A Taylor Series about $x = 0$ is called Maclaurin.</p> $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$ $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$ $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$ $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots$ $\ln(x+1) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$																												
<p style="text-align: center;">Alternating Series Error Bound</p> <p>If $S_N = \sum_{k=1}^N (-1)^k a_n$ is the N^{th} partial sum of a convergent alternating series, then</p> $ S_{\infty} - S_N \leq a_{N+1} $	<p style="text-align: center;">Geometric Series</p> $a + ar + ar^2 + ar^3 + \dots + ar^{n-1} + \dots = \sum_{n=1}^{\infty} ar^{n-1}$ <p>diverges if $r \geq 1$; converges to $\frac{a}{1-r}$ if $r < 1$</p>	<p style="text-align: center;">Some more handy INTEGRALS:</p> <p>$\int \tan x dx = \ln \sec x + C$</p> <p style="text-align: center;">$= -\ln \cos x + C$</p> <p>$\int \sec x dx = \ln \sec x + \tan x + C$</p>																												