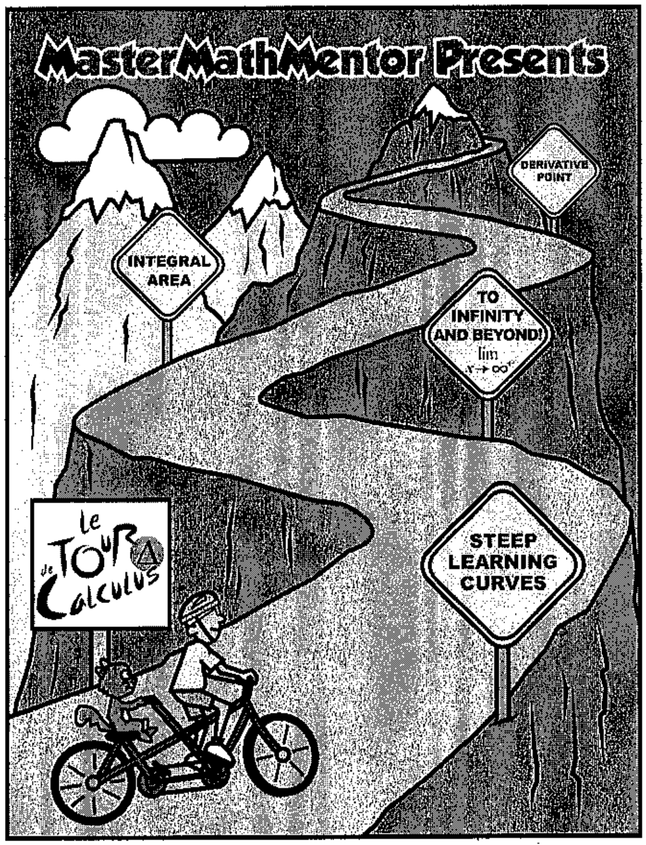
III. Precalculus Review Topics

AP Calculus Prerequisite Content



The topics presented here are important as they are used all the time in calculus. If you find that you are frequently confused by these topics, take some time to review them.

Eliminating Complex Fractions

Calculus frequently uses **complex fractions,** which are fractions within fractions. Answers are never left with complex fractions and they must be eliminated. There are two methods to eliminate complex fractions:

When the problem is in the form of $\frac{\frac{a}{b}}{\frac{c}{d}}$, we can "flip the denominator" and write it as $\frac{a}{b}\cdot\frac{d}{c}=\frac{ad}{bc}$.

However, this does not work when the numerator and denominator are not single fractions. The best way is to eliminate the complex fractions in all cases is to find the LCD (lowest common denominator) of all the fractions in the complex fraction. Multiply all terms by this LCD and you are left with a fraction that is magically no longer complex. **Important:** Note that $\frac{x^{-1}}{y^{-1}}$ can be written as $\frac{y}{x}$ but $\frac{1+x^{-1}}{y^{-1}}$ must be written as $\frac{1+\frac{1}{x}}{\frac{1}{y}}$.

• Eliminate the complex fractions.

1. $\dfrac{\frac{2}{3}}{\frac{5}{6}}$

$$\boxed{\left(\frac{\frac{2}{3}}{\frac{5}{6}}\right)\left(\frac{6}{6}\right)=\frac{4}{5}}$$

2. $\dfrac{1+\frac{2}{3}}{1+\frac{5}{6}}$

$$\boxed{\left(\frac{1+\frac{2}{3}}{1+\frac{5}{6}}\right)\left(\frac{6}{6}\right)=\frac{6+4}{6+5}=\frac{10}{11}}$$

3. $\dfrac{\frac{3}{4}+\frac{5}{3}}{2-\frac{1}{6}}$

$$\boxed{\left(\frac{\frac{3}{4}+\frac{5}{3}}{2-\frac{1}{6}}\right)\left(\frac{12}{12}\right)=\frac{9+20}{24-2}=\frac{29}{22}}$$

4. $\dfrac{1+\frac{1}{2}x^{-1}}{1+\frac{1}{3}x^{-1}}$

$$\boxed{\left(\frac{1+\frac{1}{2x}}{1+\frac{1}{3x}}\right)\left(\frac{6x}{6x}\right)=\frac{6x+3}{6x+2}}$$

5. $\dfrac{x-\frac{1}{2x}}{x^2+\frac{1}{4x^2}}$

$$\boxed{\left(\frac{x-\frac{1}{2x}}{x^2+\frac{1}{4x^2}}\right)\left(\frac{4x^2}{4x^2}\right)=\frac{4x^3-2x}{4x^4+1}}$$

6. $\dfrac{\frac{2}{5}x^{5/3}}{\frac{5}{3}}$

$$\boxed{\left(\frac{\frac{2}{5}x^{5/3}}{\frac{5}{3}}\right)\left(\frac{15}{15}\right)=\frac{6x^{5/3}}{25}}$$

7. $\dfrac{x^{-3}+x}{x^{-2}+1}$

$$\boxed{\left(\frac{\frac{1}{x^3}+x}{\frac{1}{x^2}+1}\right)\left(\frac{x^3}{x^3}\right)=\frac{1+x^4}{x+x^3}}$$

8. $\dfrac{\frac{1}{2}(2x+5)^{-2/3}}{\frac{-2}{3}}$

$$\boxed{\left(\frac{\frac{1}{2}}{\frac{-2}{3}(2x+5)^{2/3}}\right)\frac{6}{6}=\frac{-3}{4(2x+5)^{2/3}}}$$

9. $\dfrac{(x-1)^{1/2}-\frac{x(x-1)^{-1/2}}{2}}{x-1}$

$$\boxed{\left(\frac{(x-1)^{1/2}-\frac{x}{2(x-1)^{1/2}}}{x-1}\right)\left[\frac{2(x-1)^{1/2}}{2(x-1)^{1/2}}\right] \quad \frac{2(x-1)-x}{2(x-1)^{3/2}}=\frac{x-2}{2(x-1)^{3/2}}}$$

Eliminating Complex Fractions - Assignment

- Eliminate the complex fractions.

1. $\dfrac{\frac{5}{8}}{\frac{-2}{3}}$

2. $\dfrac{4-\frac{2}{9}}{3+\frac{4}{3}}$

3. $\dfrac{2+\frac{7}{2}+\frac{3}{5}}{5-\frac{3}{4}}$

4. $\dfrac{x-\frac{1}{x}}{x+\frac{1}{x}}$

5. $\dfrac{1+x^{-1}}{1-x^{-2}}$

6. $\dfrac{x^{-1}+y^{-1}}{x+y}$

7. $\dfrac{x^{-2}+x^{-1}+1}{x^{-2}-x}$

8. $\dfrac{\frac{1}{3}(3x-4)^{-3/4}}{\frac{-3}{4}}$

9. $\dfrac{2x(2x-1)^{1/2}-2x^{2}(2x-1)^{-1/2}}{(2x-1)}$

NO CALCULATOR

Limits REVIEW NAME ____________________ HOUR ____________

Evaluate the following limits. **Show your work.**

1. $\lim_{x\to\infty}\left[\ln\frac{2x+1}{x-1}\right]$

2. $\lim_{x\to 0}\left[\frac{\sin 5x}{3x}\right]$

3. $\lim_{x\to 25}\frac{\sqrt{x}-5}{x-25}$

4. $\lim_{x\to -5^-}\frac{\sqrt{(x+5)^2}}{x+5}$

5. $\lim_{x\to +\infty}\left[\frac{5-6x^4}{2x^2+4x+1}\right]$

6. $\lim_{x\to -1}\left[\frac{x^4-1}{x+1}\right]$

7. $\lim_{x\to 2}\left[\frac{x^2+x-6}{x+3}\right]$

8. $\lim_{x\to -\infty}\left(-105x^{27}+54x^{13}-75x^6-200\right)$

9. $\lim_{x\to 0}\left[\frac{x}{(2+x)^{-1}-2^{-1}}\right]$

10. $\lim_{x\to 0^+}(1-x)^{1/x}$

11. $\lim_{x\to -1}[3x^4-2x+10] =$

12. $\lim_{x\to 3}\left[\frac{x^3-x^2-5x-3}{x-3}\right] =$

13. $\lim_{x\to -\infty}\sin\left[\frac{5-2x^2}{x^4-2x^2+4}\right]$

14. $\lim_{x\to -\infty}\left[5x+2-3x^5\right] =$

15. $\lim_{x\to +\infty}\sqrt[3]{\frac{8x^7-4x^5+3x}{2x^7-x^5+1}} =$

16. $\lim_{x\to +\infty}\left[\frac{20x^{50}+20x^{44}+640}{10x^{44}+6x^{25}-320}\right] =$

17. $\lim_{x\to -\infty}\left[\frac{\sqrt{5x^2-4x}}{3x+2}\right]$

18. $\lim_{x\to -\infty}\left[\frac{5-15x^7}{3x^3+10}\right]$

19. $\lim_{x\to +\infty}\frac{4x^2+5}{3x^2+2}$

20. $\lim_{x\to 0}\left[\frac{\sin 4x}{3\sqrt{x}}\right]$

21. $\lim_{x\to -\infty}\left[\frac{5x^5+2x^2+6}{10x^7+x^3-3}\right] =$

22. $\lim_{x\to -2}\left[\frac{3x^2+x-10}{x^3+8}\right] =$

23. $\lim_{x\to 1^-}\left[\frac{x}{x^2-1}\right] =$

24. $\lim_{x\to 1}\frac{x^3-3x^2+2x}{x-1}$

25. $\lim_{x\to 0}\left[\frac{\tan 2x}{\sin ax}\right]$ $(a\neq o)$

NO CALCULATOR
Limits REVIEW

NAME ____________________ HOUR __________

26. $f(x)=\begin{cases}5-x^2, & x\le 2\\ 3x^2-4x-3, & x>2\end{cases}$

$\lim\limits_{x\to-\infty} f(x)=$

$\lim\limits_{x\to 0} f(x)=$

$\lim\limits_{x\to 2} f(x)=$

27. $\lim\limits_{x\to 2}\dfrac{1-\dfrac{4}{x^2}}{1-\dfrac{2}{x}}$

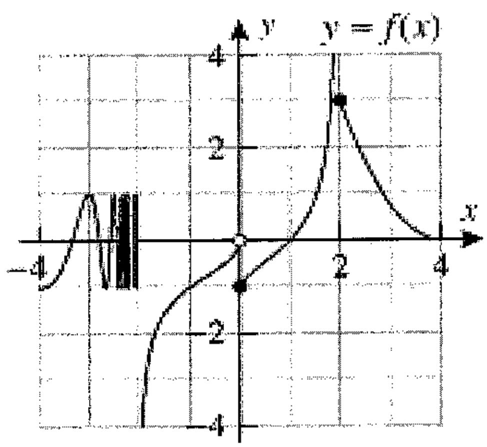
28. Find the discontinuities of. $f(x)=\dfrac{2}{2\pi-3\cos^{-1}x}$

29. Find the discontinuities of $f(x)=\dfrac{1}{1-2\cos x}$.

30. Find the values of x at which $f(x)=\dfrac{x^3+27}{x^2-9}$ is not continuous and determine if they are removable discontinuities at those points.

31. What value of k will make $g(x)$ continuous? $g(x)=\begin{cases}\frac{x^3+5x^2+x+5}{x+5}, & x\neq -5\\ k, & x=-5\end{cases}$

32.
For the function f graphed in the accompanying figure, find



(a) $\lim\limits_{x\to -2^-} f(x)$

(b) $\lim\limits_{x\to -2^+} f(x)$

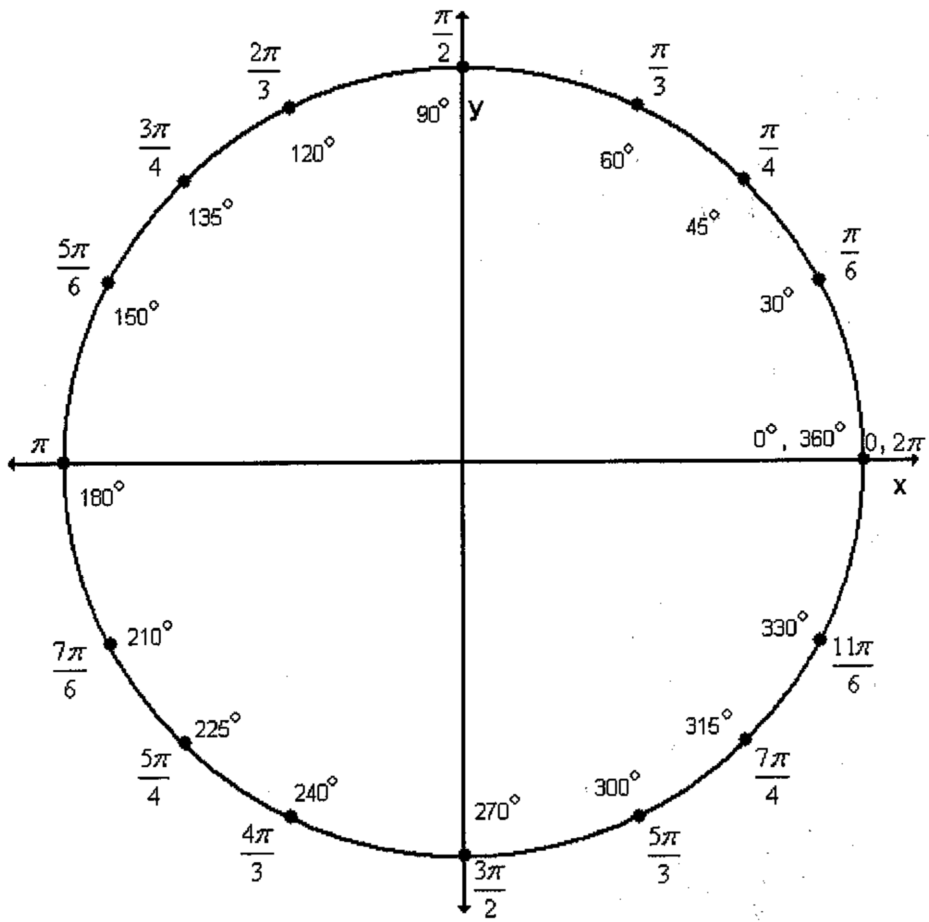
(c) $\lim\limits_{x\to 0^-} f(x)$

(d) $\lim\limits_{x\to 0^+} f(x)$

(e) $\lim\limits_{x\to 2^-} f(x)$

(f) $\lim\limits_{x\to 2^+} f(x)$

USING THE UNIT CIRCLE Memorize



Use the unit circle above to find the <u>exact</u> value of each of the following. (Exact value means no decimal approximations.) You must be able to do these in your head without re-creating the unit circle.

A) $\tan\frac{11\pi}{4} =$ **B)** $\cos\frac{5\pi}{3} =$ **C)** $\cos(-\pi) =$

D) $\sin\left(-\frac{11\pi}{6}\right) =$ **E)** $\tan\left(-\frac{10\pi}{3}\right) =$ **F)** $\csc\frac{7\pi}{3} =$

G) $\sec\left(\frac{16\pi}{3}\right) =$ **H)** $\cos\left(-\frac{11\pi}{3}\right) =$ **I)** $\sin\frac{13\pi}{4} =$

J) $\csc\left(-\frac{\pi}{6}\right) =$

K) $\tan(-3\pi) =$

L) $\cot\frac{3\pi}{2} =$

M) $\sec\left(-\frac{\pi}{3}\right) =$

N) $\cot\frac{3\pi}{4} =$

O) $\cot 20\pi =$

P) $\cos\left(-\frac{7\pi}{2}\right) =$

Q) $\sin\left(-\frac{21\pi}{4}\right) =$

R) $\cot 0 =$

S) $\sin(-4\pi) =$

T) $\cot\frac{17\pi}{3} =$

U) $\cos\frac{4\pi}{3} =$

V) Find all angles θ in the interval $[0, 2\pi)$ that satisfy the expression:

$\sin\theta = \frac{\sqrt{3}}{2}$ $\quad \theta =$ ____________________

W) Find all angles θ in the interval $[0, 2\pi)$ that satisfy the expression:

$\sec\theta = -2$ $\quad \theta =$ ____________________

X) Find all angles θ in the interval $[0, 2\pi)$ that satisfy the expression:

$\tan\theta = -1$ $\quad \theta =$ ____________________

Y) Find all angles θ in the interval $[0, 2\pi)$ that satisfy the expression:

$\csc\theta = \textit{undefined}$ $\quad \theta =$ ____________________

2) All 6 trig functions of $0, \pi/2, \pi, 3\pi/2, 2\pi$.

Exponential Functions and Logarithms

Calculus spends a great deal of time on exponential functions in the form of b^x. Don't expect that when you start working with them in calculus, your teacher will review them. So learn them now! Students must know that the definition of a **logarithm** is based on exponential equations. If $y = b^x$ then $x = \log_b y$. So when you are trying to find the value of $\log_2 32$, state that $\log_2 32 = x$ and $2^x = 32$ and therefore $x = 5$.

If the base of a log statement is not specified, it is defined to be 10. When we asked for log 100, we are solving the equation: $10^x = 100$ and $x = 2$. The function $y = \log x$ has domain $(0,\infty)$ and range $(-\infty,\infty)$. In calculus, we primarily use logs with base e, which are called natural logs (ln). So finding ln 5 is the same as solving the equation $e^x = 5$. Students should know that the value of $e = 2.71828\ldots$

There are three rules that students must keep in mind that will simplify problems involving logs and natural logs. These rules work with logs of any base including natural logs.

i. $\log a + \log b = \log(a \cdot b)$ ii. $\log a - \log b = \log\left(\frac{a}{b}\right)$ iii. $\log a^b = b \log a$

1. Find a. $\log_4 8$

$$\boxed{\begin{aligned} &\log_4 8 = x \\ &4^x = 8 \Rightarrow 2^{2x} = 2^3 \\ &x = \frac{3}{2} \end{aligned}}$$

b. $\ln\sqrt{e}$

$$\boxed{\begin{aligned} &\ln\sqrt{e} = x \\ &e^x = e^{1/2} \\ &x = \frac{1}{2} \end{aligned}}$$

c. $10^{\log 4}$

$$\boxed{\begin{aligned} &\log 4 = x \\ &10^x = 4 \text{ so } 10^{\log 4} = 4 \\ &\text{10 to a power and log are inverses} \end{aligned}}$$

d. $\log 2 + \log 50$

$$\boxed{\begin{aligned} &\log(2 \cdot 50) = \log 100 \\ &2 \end{aligned}}$$

e. $\log_4 192 - \log_4 3$

$$\boxed{\begin{aligned} &\log_4\left(\frac{192}{3}\right) \\ &\log_4 64 = 3 \end{aligned}}$$

f. $\ln\sqrt[5]{e^3}$

$$\boxed{\ln e^{3/5} = \frac{3}{5}\ln e = \frac{3}{5}}$$

2. Solve a. $\log_9\left(x^2 - x + 3\right) = \frac{1}{2}$

$$\boxed{\begin{aligned} &x^2 - x + 3 = 9^{1/2} \\ &x(x-1) = 0 \\ &x = 0, x = 1 \end{aligned}}$$

b. $\log_{36} x + \log_{36}(x-1) = \frac{1}{2}$

$$\boxed{\begin{aligned} &\log_{36} x(x-1) = \frac{1}{2} \\ &x(x-1) = 36^{1/2} = 6 \\ &x^2 - x - 6 = 0 \\ &(x-3)(x+2) = 0 \\ &\text{Only } x = 3 \text{ is in the domain} \end{aligned}}$$

c. $\ln x - \ln(x-1) = 1$

$$\boxed{\begin{aligned} &\ln\left(\frac{x}{x-1}\right) = 1 \\ &\frac{x}{x-1} = e \Rightarrow x = ex - e \\ &x = \frac{e}{e-1} \end{aligned}}$$

d. $5^x = 20$

$$\boxed{\begin{aligned} &\log\left(5^x\right) = \log 20 \\ &x \log 5 = \log 20 \\ &x = \frac{\log 20}{\log 5} \text{ or } x = \frac{\ln 20}{\ln 5} \end{aligned}}$$

e. $e^{-2x} = 5$

$$\boxed{\begin{aligned} &\ln e^{-2x} = \ln 5 \\ &-2x = \ln 5 \Rightarrow x = \frac{-\ln 5}{2} \end{aligned}}$$

f. $2^x = 3^{x-1}$

$$\boxed{\begin{aligned} &\log\left(2^x\right) = \log\left(3^{x-1}\right) \\ &x \log 2 = (x-1)\log 3 \\ &x \log 2 = x \log 3 - \log 3 \Rightarrow x = \frac{\log 3}{\log 3 - \log 2} \end{aligned}}$$

Exponential Functions and Logarithms - Assignment

1. Find a. $\log_2 \frac{1}{4}$ b. $\log_8 4$ c. $\ln \frac{1}{\sqrt[3]{e^2}}$

d. $5^{\log_5 40}$ e. $e^{\ln 12}$ f. $\log_{12} 2 + \log_{12} 9 + \log_{12} 8$

g. $\log_2 \frac{2}{3} + \log_2 \frac{3}{32}$ h. $\log_{\frac{1}{3}} \frac{4}{3} - \log_{\frac{1}{3}} 12$ i. $\log_3 \left(\sqrt{3}\right)^5$

2. Solve a. $\log_5(3x-8) = 2$ b. $\log_9\left(x^2 - x + 3\right) = \frac{1}{2}$ c. $\log(x-3) + \log 5 = 2$

d. $\log_2(x-1) + \log_2(x+3) = 5$ e. $\log_5(x+3) - \log_5 x = 2$ f. $\ln x^3 - \ln x^2 = \frac{1}{2}$

g. $3^{x-2} = 18$ h. $e^{3x+1} = 10$ i. $8^x = 5^{2x-1}$

Natural Logarithms Worksheet

Use a calculator to evaluate each expression to four decimal places.

1. $e^3 = 20.0855$

2. $e^{-2} = 0.1353$

3. $\ln 2 = 0.6931$

4. $\ln 0.09 = -2.4079$

Write an equivalent exponential or logarithmic equation.

5. $e^x = 3$

$\ln 3 = x$

6. $e^4 = 8x$

$\ln 8x = 4$

7. $\ln 15 = x$

$e^x = 15$

8. $\ln x \approx 0.6931$

$e^{0.6931} \approx x$

Evaluate each expression.

9. $e^{\ln 3} = 3$

10. $e^{\ln 2x} = 2x$

11. $\ln e^{-2.5} = -2.5$

12. $\ln e^y = y$

Solve each equation.

15. $2e^x - 1 = 11$

$2e^x = 12$

$\ln e^x = \ln 6$

$x \ln e = \ln 6$

$x = \ln 6$

$\boxed{x = 1.7918}$

17. $\overset{\ln}{e^{3x}} = \overset{\ln}{30}$

$3x(\ln e) = \ln 30$

$\frac{3x}{3} = \frac{\ln 30}{3}$

$\boxed{x = 1.1337}$

21. $\ln 3x = 2$

$\frac{e^2}{3} = \frac{3x}{3}$

$\boxed{x = 2.4630}$

20. $1 - 2e^{2x} = -19$

$-2e^{2x} = -20$

$\ln e^{2x} = \ln 10$

$2x(\ln e) = \ln 10$

$\frac{2x}{2} = \frac{\ln 10}{2}$

$\boxed{x = 1.1513}$

23. $\ln(x - 2) = 2$

$e^{2} = x - 2$

$+2 \qquad +2$

$\boxed{x = 9.3891}$

26. $\ln x + \ln 2x = 2$

$\ln x(2x) = 2$

$\ln 2x^{2} = 2$

$\frac{2x^{2}}{2} = \frac{e^{2}}{2}$

$\sqrt{x^{2}} = \sqrt{\frac{e^{2}}{2}}$

$\boxed{x = 1.9221}$

RADIOACTIVE DECAY The amount of a radioactive substance *y* that remains after *t* years is given by the equation $y = ae^{kt}$, where *a* is the initial amount present and *k* is the decay constant for the radioactive substance. If *a* = 100, *y* = 50, and *k* = -0.035, find *t*.

$y = ae^{kt}$

$\frac{50}{100} = \frac{100e^{-0.035t}}{100}$

$\ln \frac{1}{2} = \ln e^{-0.035t}$

$\ln \frac{1}{2} = \frac{-0.035t(\ln e)}{-0.035}$

$\boxed{19.8 \text{ yrs} = t}$

POPULATION In 2005, the world's population was about 6.5 billion. If the world's population continues to grow at a constant rate, the future population *P*, in billions, can be predicted by $P = 6.5e^{0.02t}$, where *t* is the time in years since 2005.

a. According to this model, what will the world's population be in 2015?

$\frac{2005 - 2015}{t = 10}$

$P = 6.5e^{0.02(10)}$

$\boxed{P = 7.94 \text{ billion}}$

b. Some experts have estimated that the world's food supply can support a population of at most 18 billion. According to this model, for how many more years will the food supply be able to support the trend in world population growth?

$P = 18$ billion

$\frac{18}{6.5} = \frac{6.5e^{0.02t}}{6.5}$

$\ln e^{0.02t} = \ln \frac{18}{6.5}$

$\frac{0.02t}{0.02} = \frac{\ln \frac{18}{6.5}}{0.02}$

$\boxed{t \approx 50.9 \text{ years}}$

Graphical Solutions to Equations and Inequalities

You have a shiny new calculator. So when are we going to use it? So far, no mention has been made of it. Yet, the calculator is a tool that is required in the AP calculus exam. For about 25% of the exam, a calculator is permitted. So it is vital you know how to use it.

There are several settings on the calculator you should make. First, so you don't get into rounding difficulties, it is suggested that you set your calculator to three decimal places. That is a standard in AP calculus so it is best to get into the habit. To do so, press $\boxed{\text{MODE}}$ and on the 2nd line, take it off FLOAT and change it to 3. And second, set your calculator to radian mode from the MODE screen. There may be times you might want to work in degrees but it is best to work in radians.

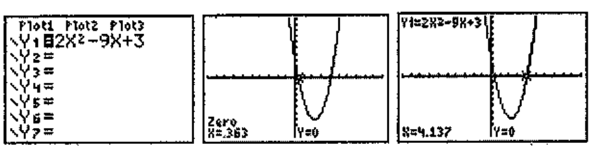
You must know how to graph functions. The best way to graph a function is to input the function using the $\boxed{\text{Y=}}$ key. Set your XMIN and XMAX using the $\boxed{\text{WINDOW}}$ key. Once you do that, you can see the graph's total behavior by pressing $\boxed{\text{ZOOM}}$ 0. To evaluate a function at a specific value of x, the easiest way to do so is to press these keys: $\boxed{\text{VARS}}$ $\boxed{\rightarrow}$ $\boxed{\text{1:Function 1}}$ $\boxed{\text{1:Y1}}$ $\boxed{(}$ and input your x-value.

Other than basic calculations, and taking trig functions of angles, there are three calculator features you need to know: evaluating functions at values of x and finding zeros of functions, which we know is finding where the function crosses the x-axis. The other is finding the point of intersection of two graphs. Both of these features are found on the TI-84+ calculator in the CALC menu $\boxed{\text{2}^{\text{ND}}}$ $\boxed{\text{TRACE}}$. They are 1:value, 2: zero, and 5: intersect.

Solving equations using the calculator is accomplished by setting the equation equal to zero, graphing the function, and using the ZERO feature. To use it, press $\boxed{\text{2}^{\text{ND}}}$ $\boxed{\text{TRACE}}$ $\boxed{\text{ZERO}}$. You will be prompted to type in a number to the left of the approximate zero, to the right of the approximate zero, and a guess (for which you can press $\boxed{\text{ENTER}}$). You will then see the zero (the solution) on the screen.

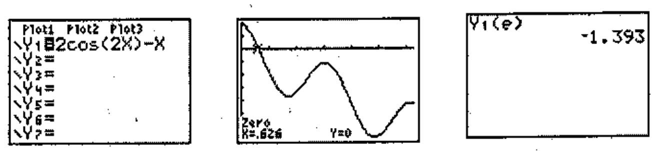
- Solve these equations graphically.

1. $2x^2 - 9x + 3 = 0$



This equation can be solved with the quadratic formula.
$$x = \frac{9 \pm \sqrt{81-24}}{4} = \frac{9 \pm \sqrt{57}}{4}$$

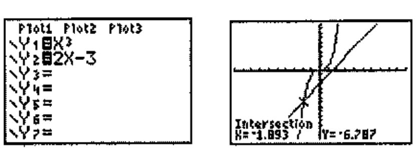
2. $2\cos 2x - x = 0$ on $[0, 2\pi)$ and find $2\cos(2e) - e$.



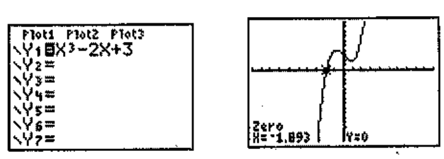
If this were the inequality $2\cos 2x - x > 0$, the answer would be $[0, 0.626)$.

3. Find the x-coordinate of the intersection of $y = x^3$ and $y = 2x - 3$

You can use the intersection feature.



Or set them equal to each other: $x^3 = 2x - 3$ or $x^3 - 2x + 3 = 0$



Graphical Solutions to Equations and Inequalities – Assignment

- Solve these equations or inequalities graphically.

1. $3x^3 - x - 5 = 0$

2. $x^3 - 5x^2 + 4x - 1 = 0$

3. $2x^2 - 1 = 2^x$

4. $2\ln(x+1) = 5\cos x$ on $[0, 2\pi)$

5. $x^4 - 9x^2 - 3x - 15 < 0$

6. $\dfrac{x^2 - 4x - 4}{x^2 + 1} > 0$ on $[0, 8]$

7. $x \sin x^2 > 0$ on $[0, 3]$

8. $\cos^{-1} x > x^2$ on $[-1, 1]$

Sigma Notation

Finding the sum of the first 5 terms of 1 + 2 + 4 + ... is ambiguous. The sum differs depending on how we interpret the numbers. $1+2+4+8+16=31$ is a logical way to interpret them with each number doubling. But $1+2+4+7+11=25$ is also logical with the difference between numbers being 1, 2, 3, and 4.

The problem with writing such addition problems with the ellipsis (...) is that the rule for each term is not apparent. We use **Sigma Notation** for such problems using the Greek letter sigma $\sum$, which means sum.

> The sum of n terms $a_1+a_2+a_3+\ldots+a_n$ is written $\sum_{i=1}^{n} a_i$ where I is the index of summation and a_i is the ith term of the sum. Sigma notation is a precise way to write a sum, but does not help to actually compute the sum.

1. $\sum_{i=1}^{8} 3$ — $3+3+\ldots+3=24$

2. $\sum_{i=1}^{6} i$ — $1+2+3+4+5+6=21$

3. $\sum_{j=1}^{7} j^2$ — $1+4+9+\ldots+49=140$

4. $\sum_{k=-2}^{3} k^3$ — $-8-1+0+1+8+27=27$

> Since $\sum_{i=1}^{n} a_i$ represents a summation of numbers, we can apply basic properties of addition and subtraction.
>
> $\sum_{i=1}^{n} ka_i = k\sum_{i=1}^{n} a_i$ (you can factor out a k). $\sum_{i=1}^{n}(a_i \pm b_i)=\sum_{i=1}^{n} a_i \pm \sum_{i=1}^{n} b_i$ (write one sum as 2 sums).

5. $\sum_{i=1}^{7} 8i = 8\sum_{i=1}^{7} i$ — $8(1+2+3+\ldots+7)$ — 224

6. $\sum_{i=1}^{9}(5i-2)=5\sum_{i=1}^{9} i-\sum_{i=1}^{9} 2$ — $5(1+2+\ldots+9)-(2+2+\ldots+2)$ — 207

7. $\sum_{i=1}^{6}(-1)^{i+1}(2i^2-1)$ — $1-7+17-31+49-71$ — -42

8. $\sum_{i=1}^{9}\frac{(-1)^i i}{i+1}$ — $-\frac{1}{2}+\frac{2}{3}-\frac{3}{4}+\ldots-\frac{9}{10}$ — -0.646

Here are some useful formulas to add many terms.

$$\sum_{i=1}^{n} k = k+k+\ldots+k = kn \qquad \sum_{i=1}^{n} i = 1+2+\ldots+n=\frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = 1^2+2^2+\ldots+n^2=\frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = 1^3+2^3+\ldots+n^3=\frac{n^2(n+1)^2}{4}$$

9. $\sum_{i=1}^{100} i = 1+2+3+\ldots+100=\frac{100(101)}{2}=5{,}050$

10. $\sum_{i=1}^{30} i^2 = 1^2+2^2+3^2+\ldots+30^2=\frac{30(31)(61)}{6}=9{,}455$

11. $\sum_{i=1}^{50} 2i^2-1=1+7+17+\ldots+4999$ — $\frac{2(50)(51)(101)}{6}-50(1)=85{,}800$

12. $\sum_{i=1}^{20} i^3-i=0+6+24+\ldots+7980$ — $\frac{20^2(21)^2}{4}-\frac{20(21)}{2}=43{,}890$

Sigma Notation – Assignment

For each problem, determine the sum of generating each term and calculate using the calculator.

1. $\sum_{i=1}^{6}(3i-2)$

2. $\sum_{j=1}^{60}10$

3. $\sum_{k=1}^{10}(k-1)^2$

4. $\sum_{k=1}^{10}k^2-1$

5. $\sum_{i=1}^{5}(i+1)(2i-3)$

6. $\sum_{i=1}^{7}\frac{i-2}{i+2}$

7. $\sum_{i=1}^{6}\frac{4}{i^2+2}$

8. $\sum_{i=1}^{25}(-1)^i 5$

9. $\sum_{i=1}^{5}\left[i^3-(i+1)^2\right]$

Use your formulas and calculators to calculate the values of the following.

10. $\sum_{i=1}^{27}6i$

11. $\sum_{i=1}^{20}i^2$

12. $\sum_{i=1}^{20}(i^2-1)$

13. $\sum_{i=1}^{35}(i+4)^2$

14. $\sum_{i=1}^{30}(i^3-i)$

15. $\sum_{i=1}^{30}(i^3-i^2)$

16. $\sum_{i=1}^{50}(2i^2-8i+4)$

17. $\sum_{i=10}^{20}i^2$

18. $\sum_{i=25}^{50}(2i^2-i)$

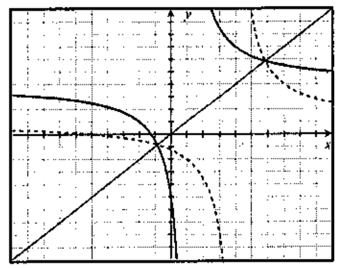
Inverses

No topic in math confuses students more than inverses. If a function is a rule that maps x to y, an **inverse** is a rule that brings y back to the original x. If a point (x,y) is a point on a function f, then the point (y,x) is on the inverse function f^{-1}. Students mistakenly believe that since $x^{-1}=\frac{1}{x}$, then $f^{-1}=\frac{1}{f}$. This is decidedly incorrect.

If a function is given in equation form, to find the inverse, replace all occurrences of x with y and all occurrences of y with x. If possible, then solve for y. Using the "horizontal-line test" on the original function f will quickly determine whether or not f^{-1} is also a function. By definition, $f\left(f^{-1}(x)\right)=x$. The domain of f^{-1} is the range of f and the range of f^{-1} is the domain of f.

1. Find the inverse to $y=\dfrac{4x+5}{x-1}$ and show graphically that its inverse is a function.

$$\boxed{\text{Inverse: } x=\frac{4y+5}{y-1}\Rightarrow xy-x=4y+5\Rightarrow y=\frac{x+5}{x-4}}$$



Note that the function is drawn in bold and the inverse as dashed. The function and its inverse is symmetrical to the line $y=x$. The inverse is a function for two reasons: a) it passes the vertical line text and b) the function passes the horizontal line test.

2. Find the inverse to the following functions and show graphically that its inverse is a function.

a. $y=4x-3$

Inverse: $x=4y-3$

$y=\dfrac{x+3}{4}$ (function)

b. $y=x^2+1$

Inverse: $x=y^2+1$

$y=\pm\sqrt{x-1}$ (not a function)

c. $y=x^2+4x+4$

Inverse: $x=y^2+4y+4$

$x=(y+2)^2\Rightarrow\pm\sqrt{x}=y+2$

$y=-2\pm\sqrt{x}$ (not a function)

3. Find the inverse to the following functions and show that $f\left(f^{-1}(x)\right)=x$

a. $f(x)=7x+4$

Inverse: $x=7y+4$

$y=f^{-1}(x)=\dfrac{x-4}{7}$

$f\left(\dfrac{x-4}{7}\right)=7\left(\dfrac{x-4}{7}\right)+4=x$

b. $f(x)=\dfrac{1}{x-1}$

Inverse: $x=\dfrac{1}{y-1}\Rightarrow xy-x=1$

$y=f^{-1}(x)=\dfrac{x+1}{x}$

$f\left(\dfrac{x+1}{x}\right)=\left(\dfrac{1}{\frac{x+1}{x}-1}\right)\left(\dfrac{x}{x}\right)$

$=\dfrac{x}{x+1-x}=x$

c. $f(x)=x^3-1$

Inverse: $x=y^3-1$

$y=f^{-1}(x)=\sqrt[3]{x+1}$

$f\left(\sqrt[3]{x+1}\right)=\left(\sqrt[3]{x+1}\right)^3-1=x$

4. Without finding the inverse, find the domain and range of the inverse to $f(x)=\sqrt{x+2}+3$.

Function: Domain: $[-2,\infty)$, Range: $[3,\infty)$ Inverse: Domain: $[3,\infty)$, Range: $[-2,\infty)$

Inverses – Assignment

1. Find the inverse to the following functions and show graphically that its inverse is a function.

a. $2x-6y=1$

b. $y=ax+b$

c. $y=9-x^2$

d. $y=\sqrt{1-x^3}$

e. $y=\dfrac{9}{x}$

f. $y=\dfrac{2x+1}{3-2x}$

2. Find the inverse to the following functions and show that $f\left(f^{-1}(x)\right)=x$

a. $f(x)=\dfrac{1}{2}x-\dfrac{4}{5}$

b. $f(x)=x^2-4$

c. $f(x)=\dfrac{x^2}{x^2+1}$

3. Without finding the inverse, find the domain and range of the inverse to $f(x)=\dfrac{\sqrt{x+1}}{x^2}$.