

CHAPTER 4 TEST REVIEW

4.1 Implicit Differentiation

1. Find $\frac{dy}{dx}$. $\sin(x^2y) = y^2 + \cos(2x)$
2. Find $\frac{d^2y}{dx^2}$. $4x^2 + 5y^2 = 10$
3. Find the equation of the line tangent to the ellipse $x^2 + 4y^2 = 16$ at the point $\left(3, \frac{-\sqrt{7}}{2}\right)$
4. Find $\frac{dy}{dx}$. $\sin(2y) = y^2 + \tan^{-1}(2x)$
5. Find $\frac{dy}{dx}$. $x^3y^2 - 5x^2y + x = 1$

4.2 Derivatives of Logs

In 6 – 10, find $\frac{dy}{dx}$.

6. $y = \ln(\cos x)$
7. $y = \log_4(6x^2 + 2)$
8. $y = \ln \left[\frac{\tan^4 x \cdot \sqrt[3]{5x+1}}{(5x^2+2)^5} \right]$
9. $y = \frac{\sqrt{x^2 + 5} \cdot \cos^3 x}{(e^{5x})^2}$
10. Find the instantaneous rate of change of y with respect to x at $x = 0$ of the equation if $y = \ln[\cos^2 x \cdot \sqrt{1-3x}]$

4.3 Derivatives of Inverse Functions

11. Find the derivative of $f^{-1}(x)$ if $f(x) = 8x^3 - 4x^2 + 2x - 5$.

4.3 Derivatives Exponential Functions

In 12 – 15, find $\frac{dy}{dx}$.

12. $y = e^{\sqrt{2x+1}}$
13. $y = e^{(3x^2-2)}$
14. $y = 2^{\sin x}$
15. $y = 2^{-x}$
16. At what value of x does $y = 5^x$ have a tangent line with slope equal to $\ln(5)$?

4.3 Derivatives Inverse Trig Functions

In 17 - 20, find $\frac{dy}{dx}$.

17. $y = \sec^{-1}(4x^2)$

18. $y = \tan^{-1}\left(\frac{2}{x}\right)$

19. $y = \cos^{-1}(5 - x)$

20. $y = x^{\sin^{-1}x}$

21. $y = [\cot^{-1}x]^2$

ANSWERS:

1. $y' = \frac{-2 \sin(2x) - 2xy \cos(x^2y)}{x^2 \cos(x^2y) - 2y}$

2. $y'' = \frac{-8}{5y^3}$

3. $y + \frac{\sqrt{7}}{2} = \frac{3}{2\sqrt{7}}(x - 3)$

4. $y' = \frac{1}{1+4x^2} \cdot \frac{1}{\cos(2y)-y}$

5. $y' = \frac{-3x^2y^2 + 10xy - 1}{2x^3y - 5x^2}$ or $y' = \frac{3x^2 - 10xy + 1}{5x^2 - 2x^3y}$

6. $y' = -\tan(x)$

7. $y' = \frac{6x}{(3x^2+1)\ln 4}$

8. $y' = 4\sec x \csc x + \frac{5}{3(5x+1)} - \frac{50x}{5x^2+2}$

9. $y' = \left(\frac{x}{x^2+5} - 3\tan x - 10\right) \frac{\sqrt{x^2+5} \cdot \cos^3 x}{(e^{5x})^2}$

10. $\frac{-3}{2}$

11. $y' = \frac{1}{24y^2 - 8y + 2}$

12. $y' = \frac{e^{\sqrt{2x+1}}}{\sqrt{2x+1}}$

13. $y' = 6xe^{3x^2-2}$

14. $y' = 2^{\sin x} \cdot \ln 2 \cdot \cos x$

15. $y' = \frac{-\ln 2}{2^x}$

16. $x = 0$

17. $y' = \frac{2}{|x|\sqrt{16x^4-1}}$

18. $y' = \frac{-2}{x^2+4}$

19. $y' = \frac{1}{\sqrt{1-(5-x)^2}}$ or $y' = \frac{1}{\sqrt{-24+10x-x^2}}$ Not necessary because it doesn't get your anywhere

20. $y' = x^{\sin^{-1}x} \left[\frac{\ln x}{\sqrt{1-x^2}} + \frac{\sin^{-1}x}{x} \right]$

21. $y' = \frac{-2\cot^{-1}x}{1+x^2}$