

AP Calc Chapter 6 Review

1. Evaluate the indefinite integral.

$$\int \frac{3}{4x} dx = \frac{3}{4} \ln|x| + C$$

$$\int \sin x + \pi dx = -\cos x + \pi x + C$$

$$\int \frac{4}{\sqrt{1-x^2}} + \sec^2 x dx = 4\sin^{-1}(x) + \tan x + C$$

$$\int x^2 \sqrt{x-2} dx = \frac{2}{7}(x-2)^{7/2} + \frac{8}{5}(x-2)^{5/2} + \frac{8}{3}(x-2)^{3/2} + C$$

2. Evaluate the definite integral.

$$\int_{\ln 2}^{\ln 5} e^x dx = 3$$

$$\int_3^{-3} \frac{12+x^2}{x^2} dx = 2$$

$$\int_0^4 |x^2 - 4x + 3| dx = 4$$

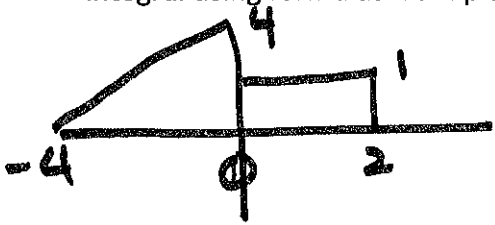
$$\int_{\pi/2}^{3\pi/2} f(x) dx \text{ if } f(x) = \begin{cases} \sin x, & x < \pi \\ \cos x, & x \geq \pi \end{cases} = 0$$

$$\int_4^4 x^2 + 2x + 2 dx = 0$$

$$\int_0^{\pi/2} \sin^2 3x \cos 3x dx = -\frac{1}{9}$$

$$\int_2^4 x\sqrt{x-2} dx = \frac{64\sqrt{5}}{15} \approx 6.03397$$

3. Let $f(x) = \begin{cases} 1 & x \geq 0 \\ x+4 & x < 0 \end{cases}$. Sketch and give a geometric interpretation for $\int_{-4}^2 f(x) dx$ and evaluate this integral using formulas from plane geometry and using Calculus.



$$\int_{-4}^0 (x+4) dx + \int_0^2 1 dx$$

$$8 + 2 = 10$$

4. Write the integral that represents the net signed area between the curve, $f(x) = x^3 - 6x^2 + 8x$ and the x -axis over the interval $[2, 6]$. Evaluate your integral using your calculator.

$$\int_2^6 (x^3 - 6x^2 + 8x) dx = 32$$

5. Find the total area between the curve, $f(x) = x^3 - 6x^2 + 8x$ and the x -axis over the interval $[2, 6]$.

$$\int_2^6 |x^3 - 6x^2 + 8x| dx = 40$$

6. A particle moves along an s -axis with acceleration $a(t) = \sin(t)$ if $v(0) = -1$ and $s(0) = 1$. Find the total distance traveled by the particle over the interval $0 \leq t \leq \frac{3\pi}{2}$ and the position function. Find $s\left(\frac{\pi}{4}\right)$.

$$v(t) = -\cos(t)$$

$$s(t) = -\sin(t) + 1$$

$$s\left(\frac{\pi}{4}\right) = -\sin\left(\frac{\pi}{4}\right) + 1$$

$$= -\frac{\sqrt{2}}{2} + 1$$

7. $F(x) = \int_3^x \frac{t}{t-4} dt$

a. Find $F(3)$. 0

b. Find $F'(3)$. -3

c. Find $F''(3)$. -4

d. Over what intervals is $F(x) = \int_3^x \frac{t}{t-4} dt$ increasing or decreasing?

e. Describe the concavity of $F(x) = \int_3^x \frac{t}{t-4} dt$

INC: $(-\infty, 0) \cup (4, +\infty)$
 DEC: $(0, 4)$

Concave down $(-\infty, +\infty)$

8. $\frac{dy}{dx} = x\sqrt{4-x^2}$ Solve the differential equation with initial condition $y(2) = 3$.

$$y = -\frac{1}{3}(4-x^2)^{3/2} + 3$$

9. $\frac{dy}{dx} = x\sqrt{x^2+4}$ Solve the differential equation with initial condition $y(0) = \frac{-1}{3}$.

$$y = \frac{1}{3}(x^2+4)^{3/2} - 3$$