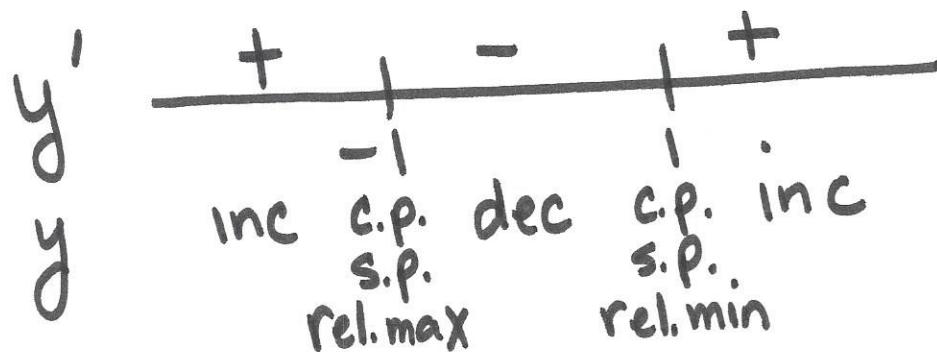


5.1 - 5.4 Test Review

① $y' = 3x^2 - 3$ $0 = 3x^2 - 3$
 $x = \pm 1$

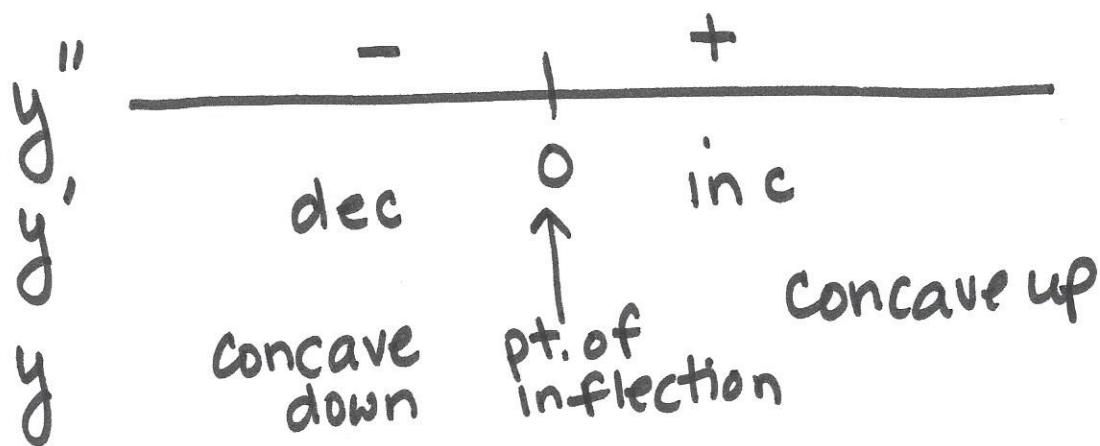


INC: $(-\infty, -1) \cup (1, +\infty)$
 DEC: $(-1, 1)$

$$y'' = 6x$$

$$6x = 0$$

$$x = 0$$



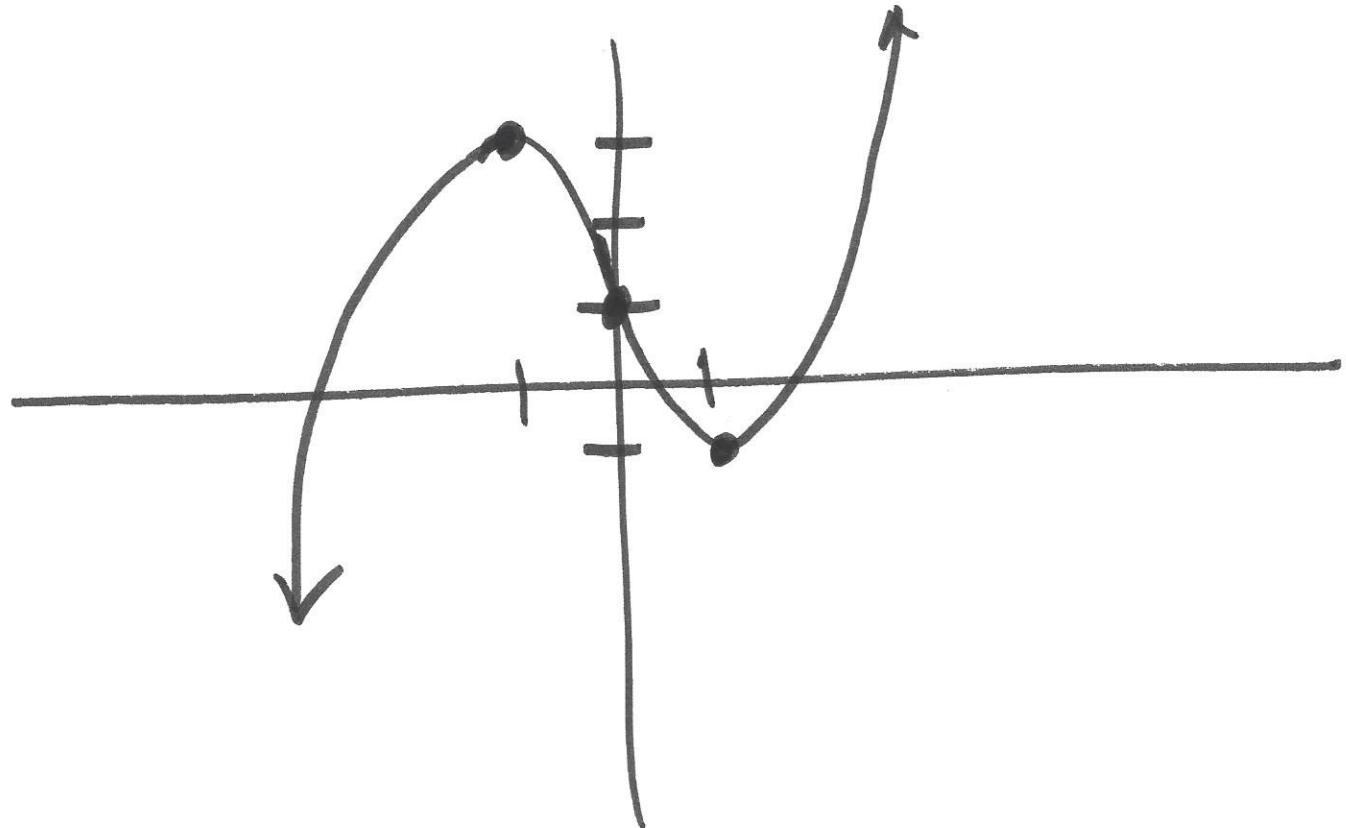
concave down: $(-\infty, 0)$

concave up: $(0, +\infty)$

① Sketch $y = x^3 - 3x + 1$

Things we know:

- continuous everywhere
- $\downarrow \uparrow$ end behavior
- $(0, 1)$ is a y-intercept
- Rel. max @ $x = -1$ $(-1, 3)$
 $\rightarrow (-1)^3 - 3(-1) + 1 = -1 + 3 + 1 = 3$
- Rel. min @ $x = 1$ $(1, -1)$
- pt of inflection @ $x = 0$ $(0, 1)$
- Intervals of inc/dec (f' sign chart)
- Intervals of concavity (f'' sign chart)



② $f'(x) = 6x^2 - 6x - 12$ on closed interval $[-2, 2]$

$$0 = 6x^2 - 6x - 12$$

$$0 = 6(x^2 - x - 2)$$

$$0 = 6(x-2)(x+1)$$

$$x = 2 \text{ or } -1$$

Note: relative extrema cannot happen at endpt.

$$f' \begin{array}{c} [+ | -] \\ \hline -2 \quad -1 \quad \frac{1}{2} \end{array} +$$

rel. max rel. min

5.4 Absolute extrema - evaluate $f(x)$ at all critical points and at endpoints. The largest is the absolute max and the lowest is the absolute min.

$$f(-2) = 2(-2)^3 - 3(-2)^2 - 12(-2) + 8 = 4$$

$$f(-1) = 15 \text{ largest}$$

$$f(2) = -12 \text{ smallest}$$

Relative max/Absolute max @ $x = -1$ $(-1, 15)$

Absolute min @ $x = 2$ $(2, -12)$

* On the interval $(-\infty, \infty)$:

Relative max @ $x = -1$

Relative min @ $x = 2$

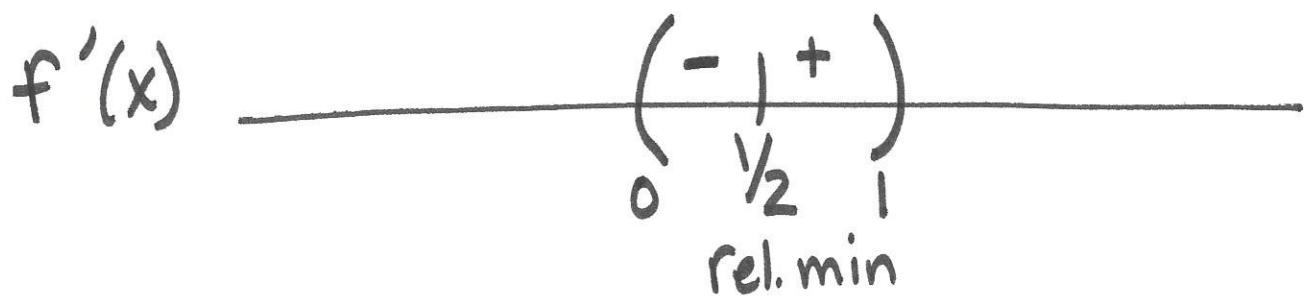
No Absolute extrema

$$③ \quad f'(x) = \frac{-(1-2x)}{(x-x^2)^2} \quad * (0,1) \text{ interval}$$

From $f(x)$, $x \neq 0$ and $x \neq 1$ V.A.
 $\therefore x=0$ and $x=1$ are NOT c.p.

However, there is a s.p. when

$$0 = -(1-2x) \quad @ x = \frac{1}{2}$$



5.4 Absolute Extrema on an open interval may or may not have absolute extrema. On an open interval (a,b) you must evaluate $\lim_{x \rightarrow a^+} f(x)$, rel. extrema, and $\lim_{x \rightarrow b^-} f(x)$.

$$\lim_{x \rightarrow 0^+} \frac{1}{x-x^2} = \frac{1}{0} = \text{DNE} \quad (\text{Test } x=0 \quad +\#)$$

$$= +\infty$$

$$f\left(\frac{1}{2}\right) = \frac{1}{\frac{1}{2} - \frac{1}{4}} = \frac{1}{\frac{1}{4}} = 4 \text{ rel. min}$$

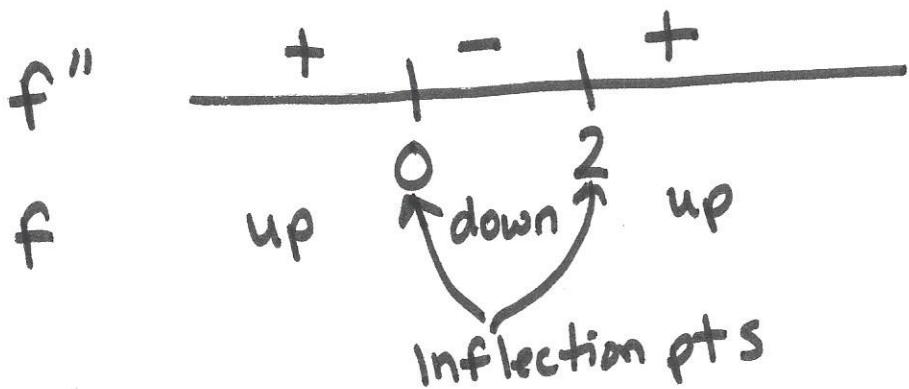
$$\lim_{x \rightarrow 1^-} \frac{1}{x-x^2} = \text{DNE} = +\infty$$

No absolute extrema

(4.)

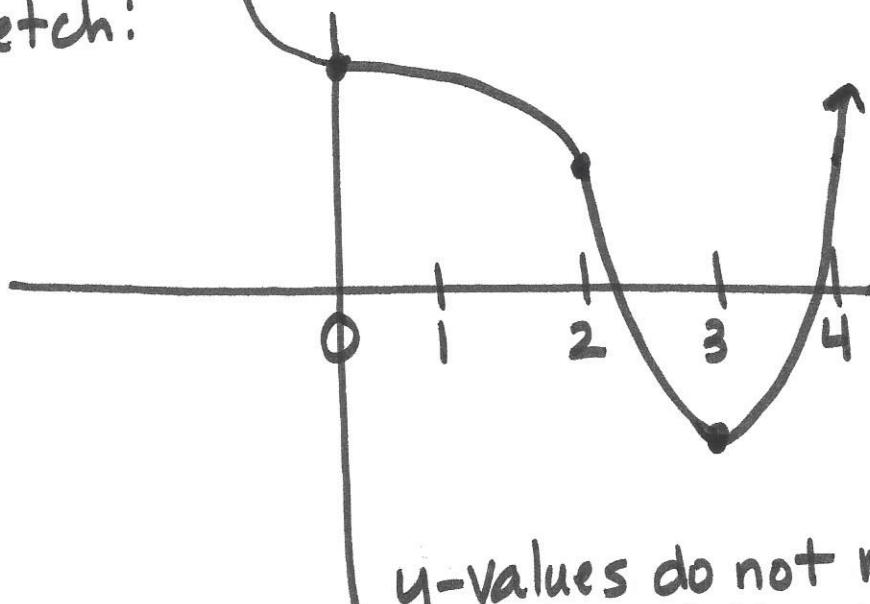
f'	-	+	-	+	+
f	dec	s.p.	dec	s.p.	inc

rel. min



Note: $x=0$ is a critical point/stationary point but not relative extrema. Relative extrema happen at stationary points but not all s.p. are extrema.

Sketch:



y-values do not matter, we do not know $f(x)$.

$$\textcircled{6.} \quad f' \begin{array}{c} + - - \\ \hline 0 | 1 | 3 | 5 \end{array}$$

- a) Rel. max @ $x=1$ because
 $f'(x)=0$ and $f'(x)$ changes
 from pos. to neg.
 No rel. mins.

b)

$$f'' \begin{array}{c} - + - + \\ \hline 0 | 2 | 3 | 4 | 5 \end{array}$$

$f(x)$ has inflection points at $x=2, 3, +4$
 because $f'(x)$ is defined which
 means $f(x)$ is continuous at those
 values and $f''(x)$ changes signs.

Note: $f''(x) \neq 0$ at 3, so you cannot use
 that as a justification.

(7)

$$f'(x) = 12x - 3x^2$$

$$0 = 3x(4-x)$$

$$x=0 \quad + \quad x=4 \quad S.P.$$

$$\begin{array}{c} f' \\ \hline - | + | - \\ 0 \qquad \qquad \qquad 4 \end{array}$$

- $f(x)$ has a relative min. @ $x=0$
because $f'(x)=0$ and changes
from neg. to pos.

- $f(x)$ has a relative max. @ $x=4$
because $f'(x)=0$ and changes
from pos. to neg.

$$\begin{array}{c} f'' \\ \hline + | - \\ \hline 2 \end{array}$$

$$f''(x) = 12-6x$$

$$0 = 12-6x$$

$$-12 = -6x$$

$$x = 2$$

$f(x)$ has a pt. of inflection
@ $x=2$ because $f''(x)=0$
and changes signs.

or because $f(x)$ is continuous
at $x=2$ and $f''(x)$ changes
signs at that x -value.

⑧ V.A. @ $x = \pm 4$ so they cannot be critical points.

$$f'(x) = \frac{-34x}{(x^2 - 16)^2}$$

$$\begin{array}{c} f' \\ \hline + \quad + \quad | \quad - \quad - \end{array} \quad \begin{array}{c} -4 \quad 0 \quad 4 \end{array}$$

INC: $(-\infty, 0)$ DEC: $(0, +\infty)$
 because $f'(x) > 0$ because $f'(x) < 0$

⑨ a) $\lim_{x \rightarrow +\infty} e^x \cdot x^{2/3} = +\infty$ Pg. 130

$$\lim_{x \rightarrow -\infty} e^x \cdot x^{2/3} = 0$$

b) $0 = e^x \cdot x^{2/3}$
 $e^x = 0 \quad x^{2/3} = 0$ $(0, 0)$ x-int
 Never $x = 0$

c) $y = e^0 \cdot 0^{2/3} = 0 \quad (0, 0)$ y-int

⑨ cont.,

d.) $y' = \frac{2e^x}{3x^{1/3}} + x^{2/3}e^x$

$$y' = e^x \left(\frac{2}{3x^{1/3}} + x^{2/3} \right)$$

$$y' = e^x \left(\frac{2+3x}{3x^{1/3}} \right)$$

$$e^x = 0$$

Never

$$2+3x=0$$

$$x = -\frac{2}{3}$$

s.p.

$$3x^{1/3}=0$$

$$x=0$$

c.p.

f'	+	-	+	
f	inc	$-\frac{2}{3}$ dec	0	inc

s.p. c.p.
 rel. max rel. min
 cusp

INC: $(-\infty, -\frac{2}{3}) \cup (0, +\infty)$

DEC: $(-\frac{2}{3}, 0)$

c.) *Need a calculator for this

$$e^x = 0$$

Never

$$9x^2 + 12x - 2 = 0$$

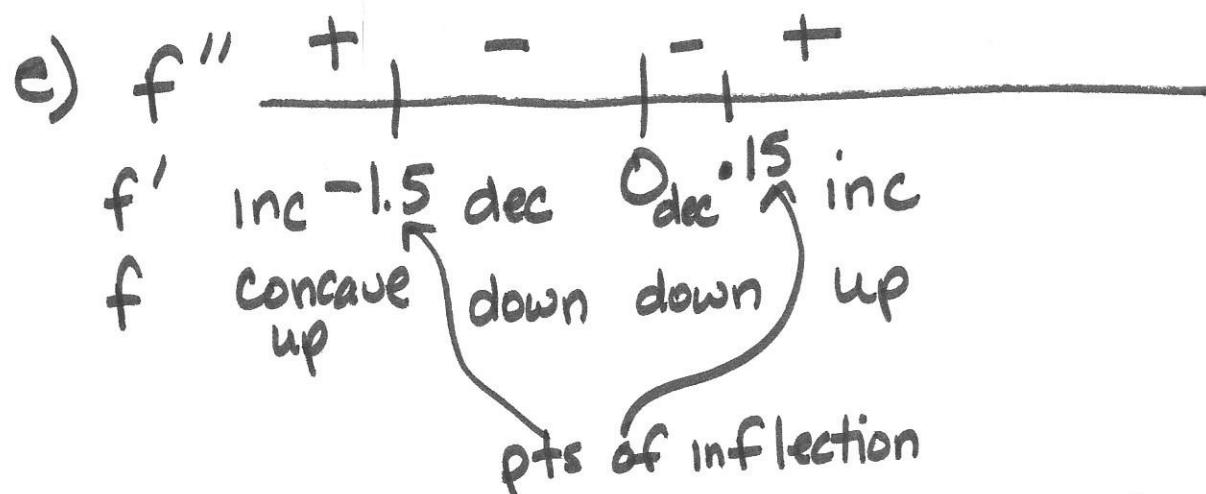
$$x \approx -1.5$$

$$x \approx .15$$

$$9x^{4/3} = 0$$

$$x = 0$$

⑨. cont., Use calculator



Concave up: $(-\infty, -1.5) \cup (.15, +\infty)$

Concave down: $(-1.5, .15)$

f. Sketch *DO NOT USE A CALCULATOR*

