Skill Builder: Topic 3.6 – Higher-Order Derivatives

Find the indicated derivative of each function.

1.) Find y'' if $y=4x^6+3x^5-8x^3+6x^2+15$ $y'(x)=24x^5+15x^4-24x^2+12x$ $y''(x)=120x^4+60x^3-48x+12$	**2.)** What is $\frac{d^2y}{dx^2}$ when $y=\cos\left(2x^3\right)$ $\frac{dy}{dx}=-\sin(2x^3)\cdot(6x^2)=-6x^2\cdot\sin(2x^3)$ Now, to find $\frac{d^2y}{dx^2}$, we must use the Product Rule. $\frac{d^2y}{dx^2}=-12x\cdot\sin(2x^3)+(-6x^2)\cdot\cos(2x^3)\cdot(6x^2)$ $=-12x\cdot\sin(2x^3)-36x^4\cdot\cos(2x^3)$
3.) Find $y^{(4)}$ if $y=3x^3+2x^2-x+9$ $y'(x)=9x^2+4x-1$ $y''(x)=18x+4$ $y'''(x)=18$ $y^{(4)}(x)=0$	**4.)** The second derivative of $f(x)=\ln x$ at $x=3$ is what value? $f'(x)=\frac{1}{x}$ $f''(x)=-\frac{1}{x^2}$ $f''(3)=-\frac{1}{9}$
5.) If $f(x)=\sqrt{x-16}$, what is $f''(x)$? $f(x)=(x-16)^{1/2}$ $f'(x)=\frac{1}{2}(x-16)^{-1/2}(1)$ $f''(x)=-\frac{1}{4}(x-16)^{-3/2}(1)$ $=\frac{-1}{4\sqrt{(x-16)^3}}$	**6.)** A function g is defined by $g(x)=3e^{3x}$, what is $g''(2)$? $g'(x)=3e^{3x}\cdot 3=9e^{3x}$ $g''(x)=9e^{3x}\cdot 3$ $=27e^{3x}$ $g''(2)=27e^{2(3)}=27e^6$
7.) If $f(x)=(2+3x)^4$, find the fourth derivative of f. $f'(x)=4(2+3x)^3\cdot 3=12(2+3x)^3$ $f''(x)=36(2+3x)^2\cdot 3=108(2+3x)^2$ $f'''(x)=108(2+3x)^1\cdot 3=324(2+3x)$ $f^{(4)}(x)=324\cdot 3=972$	**8.)** What is the 20th derivative of $y=\sin(2x)$? $y'=\cos(2x)\cdot 2=2\cos(2x)$ $y''=-2\sin(2x)\cdot 2=-4\sin(2x)$ Note: $2^2=4$ $y'''=-4\cos(2x)\cdot 2=-8\cos(2x)$ Note: $2^3=8$ $y^{(4)}=8\sin(2x)\cdot 2=16\sin(2x)$ Note: $2^4=16$ $y^{(5)}=16\cos(2x)\cdot 2=32\cos(2x)$ Note: $2^5=32$ Notice how the fifth derivative returns to some constant times $\cos(2x)$. This means that the 20th derivative will be similar to the 4th, 8th, 12th, etc derivatives. The only difference is that the leading coefficient will be 2^{20}. $\therefore y^{(20)}=2^{20}\sin(2x)$
9.) If $y=xe^x$, then find $\frac{d^ny}{dx^n}$. $\frac{dy}{dx}=1\cdot e^x+x\cdot e^x=e^x+xe^x$ $\frac{d^2y}{dx^2}=e^x+1\cdot e^x+x\cdot e^x=2e^x+xe^x$ $\frac{d^3y}{dx^3}=2e^x+1\cdot e^x+x\cdot e^x=3e^x+xe^x$ $\vdots$ $\frac{d^ny}{dx^n}=n\cdot e^x+xe^x$	**10.)** Find $h''(x)$ if $h(x)=f(x^3)$. $h'(x)=f'(x^3)\cdot x^2$ The next derivative requires the Product Rule. $h''(x)=f''(x^3)\cdot x^2\cdot x^2+f'(x^3)\cdot 2x$ $=x^4f''(x^3)+2xf'(x^3)$