CHA		
1	Topic: 4.2	Straight-Line Motion: Connecting Position, Velocity and Acceleration
Learning Objective CHA-3.B: Calculate rates of change in applied contexts.		

Just as you can obtain a velocity function from differentiating a position function, you can obtain an **acceleration** function by differentiating a velocity function.

$s(t)$ or $x(t)$	Position function
$v(t) = s'(t)$	Velocity function
$a(t) = v'(t) = s''(t)$	Acceleration function

When we study motion initially in calculus, we consider what is referred to as **straight-line motion**. When we study this idea, we are concerned with four concepts: *position, velocity, speed*, and *acceleration.*

Position

Notation for a position function with respect to time t is usually $s(t)$ or $x(t)$ if the object is moving along the x-axis and $y(t)$ if the object is moving along the y-axis.

Example 1: For $s(t) = t^2 - 2t - 3$, show its position on the number line for $t = 0,1,2,3,4$.

Velocity

When an object moves, its position changes over time. So, we can say that the velocity function, $v(t)$ is the change of the position function over time. We know this to be the derivative and can thus say that $v(t)=s'(t)$. For convenience sake, we will define $v(t)$ in the following way:

Motion	$v(t) > 0$	$v(t) < 0$	$v(t) = 0$
Horizontal Line	object moves to the **right**	object moves to the **left**	object **stopped**
Vertical Line	object moves **up**	object moves **down**	object **stopped**

Speed is not synonymous with velocity. Speed does not indicate direction. So, we define the speed function:

Speed $= |v(t)|$

The **speed** of an object must either be positive or zero (meaning the object has stopped).

Acceleration

The definition of acceleration is the change in velocity over time. We know this to be a derivative and can thus say that $a(t)=v'(t)=s''(t)$. So, given a position function $s(t)$, we can now determine both the **velocity** and **acceleration** function.

On your cars, you have two devices that change **velocity**. What are they? The ______________________ and the ______________________. For convenience sake, let us define the **acceleration** function like this:

Motion	$a(t) > 0$	$a(t) < 0$	$a(t) = 0$
Horizontal Line	object accelerating to the **right**	object accelerating to the **left**	velocity not changing
Vertical Line	object accelerating **upwards**	object accelerating **downwards**	velocity not changing

Just because an object's acceleration is zero does not mean that the object is stopped. It means that the velocity is not changing.

What device on your car will keep the car's acceleration equal to zero? ________________________________

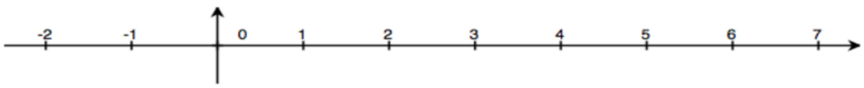
Also, just because you have a positive acceleration does not mean that you are moving to the right. For instance, suppose you were walking to the right [$v(t) > 0$], when all of a sudden, a large wind started to blow to the left [$a(t) < 0$]. What would occur? __________________________________

<u>Example 2</u>: A particle moves along the *x*-axis with position function $s(t) = t^2 - 4t + 2$.

$v(t) =$ ______________________________ $a(t) =$ ______________________________

Complete the chart for the first 5 seconds and show where the particle is on the number line.

t	$s(t)$	$v(t)$	$\lvert v(t) \rvert$	$a(t)$	Description
0					
1					
2					
3					
4					
5					



The work we did in Example 2 is a bit much. Luckily, we can simplify it. In general, we are concerned with **i)** what direction the particle is moving, **ii)** when it is stopped, and **iii)** when it is speeding up or slowing down.

The Relationship Between Velocity and Acceleration

Fill in each box with either of the phrases: "speeding up," "slowing down," "constant speed," or "stopped."

How are we moving?	$a(t) > 0$	$a(t) < 0$	$a(t) = 0$
$v(t) > 0$			
$v(t) < 0$			
$v(t) = 0$			

To determine intervals where particles are slowing down and speeding up, we generate a sign chart for velocity, a sign chart for acceleration and a 3rd sign chart (which we'll call motion), which is the product of the first two.

<u>Example 3</u>: Determine the time intervals where particles traveling on a straight line, whose position is defined by $s(t) = t^3 - 3t^2$, are slowing down, or speeding up.

On the AP Exam, students are frequently asked to find the time intervals when the particle is speeding up and slowing down and also asked to "justify their answers." Sign charts are not scored as justifications by the readers. Instead, the readers are looking for an answer in sentence form. While students are encouraged to make sign charts (because they are easy to construct), the justification must be written a bit more formally and reference the comparison of the signs of $v(t)$ and $a(t)$ over an interval.

Example 4: A particle is moving along a horizontal line with position function $s(t) = t^3 - 9t^2 + 24t + 4$. Do an analysis of the particle's direction, acceleration, motion (speeding up or slowing down), and position.

$v(t)$

0

$a(t)$

0

motion

0

position

Motion Affected by Gravity

Although this concept is not tested on the AP Calculus Exam, it provides a great environment in which we can understand particle motion. When an object is subjected to gravity, we can determine its position, velocity, and acceleration based on the formulas to the right.
Note that v_0 is the initial velocity $v(0)$ and that a_0 is the initial acceleration $a(0)$.

	feet measurement	meter measurement
position $s(t)$	$-16t^2 + v_0t + s_0$	$-4.9t^2 + v_0t + s_0$
velocity $v(t)$	$-32t + v_0$	$-9.8t + v_0$
acceleration $a(t)$	-32	-9.8

Example 5: A rock thrown vertically upward from the surface of the moon at a velocity of 24 m/sec reaches a height of $s(t) = -0.8t^2 + 24t$ meters in t seconds.

a.) Find the rock's velocity and acceleration as a function of time.	**b.)** How long will it take the rock to reach its highest point?
c.) How high did the rock go?	**d.)** How long did it take the rock to reach half its maximum height?
e.) How long was the rock aloft?	**f.)** Find the rock's speed when hitting the surface of the moon.

Example 6: A particle moves along the x-axis with position function, $x(t) = sin(e^{0.5t})$. Determine for which of the integer values $t = 1, 2, 3, 4, 5$, is the particle both to the right of the y-axis and is speeding up? Explain your reasoning.