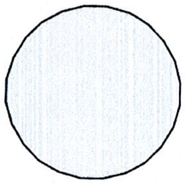
CHA	AP CALCULUS AB	
1	Topic: 4.6	Approximating Values of a Function Using Local Linearity and Linearization

Learning Objective CHA-3.F: Approximate a value on a curve using the equation of a tangent line.

Curvature as Consecutive Lines



Look carefully at the figure to the left. What do you see?

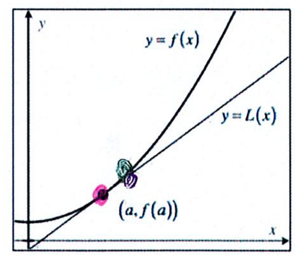
While the figure may appear to be a circle, it is actually a 24-sided regular polygon. (In case you ever appear on Jeopardy, its technical name is an *icosikaitera*.) When the number of sides become this large, the length of the sides become smaller.

The calculus definition of a circle is simply a polygon whose number of sides approaches ∞ and who lengths of sides approach 0.

The above phenomenon occurs quite a bit in real life. Examine guardrails on the side of a curving road and you will see that they do not match the curve of the road but in reality, are a series of straight-line rails. Since the straight-line sections of railing are numerous, they appear to curve.

A wooden roller coaster is built the exact same way. It is far too difficult to create curved wooden side rails so straight planks of wood are used and when there are many of them, they can simulate the curvature of the coaster's hills.

Linear Approximations



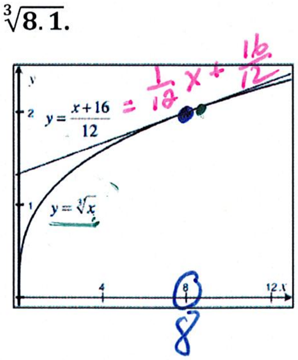
Suppose we have a function $f(x)$ and its tangent line drawn at the point $(a, f(a))$ as shown in the figure to the right. The equation of the tangent line, which we will call $L(x)$, is given by the equation:

$L(x) - f(a) = f'(a)(x - a)$ or $L(x) = f'(a)(x - a) + f(a)$

$y - y_1 = m(x - x_1)$ $\quad$ $y = m(x - x_1) + y_1$

Near $x = a$, the function and the tangent line have nearly the same graph. If k were some x-value very near to a, there would be very little difference in the values of $L(k)$ and $f(k)$. In this case, we call the tangent line the **linear approximation** to the function at $x = a$.

Example 1: Use the linear approximation for $\boldsymbol{f(x) = \sqrt[3]{x}}$ **at** $\boldsymbol{x = 8}$ to approximate $\boldsymbol{\sqrt[3]{8.1}}$.



Scan the QR Code above to watch a video covering Example 1

$f'(x) = \frac{1}{3}x^{-2/3}$ $\quad$ $f(x) = x^{1/3}$ $\quad y = 2$

$f'(8) = \frac{1}{3(8)^{2/3}} = \frac{1}{12}$

$y - 2 = \frac{1}{12}(x - 8)$

$L(x) = \frac{1}{12}(x - 8) + 2$

$L(8.1) = \frac{1}{12}(8.1 - 8) + 2$

$= \frac{1}{12}\left(\frac{1}{10}\right) + 2 \approx 2 + \frac{1}{120} \approx \sqrt[3]{8.1}$ over-estimate

Example 2: Using a calculator, find the error in using the linear approximation to $f(x) = e^x$ at $x = 0$ to approximate $\sqrt[4]{e}$ $= e^{1/4}$

$y = 1$

Scan the QR Code above to watch a video covering Example 2

$f'(x) = e^x$

$f'(0) = e^0 = 1$

$y - 1 = 1(x - 0)$

$L(x) = x + 1$

$L\left(\frac{1}{4}\right) = \frac{1}{4} + 1 = 1.25$

$\sqrt[4]{e} \approx 1.25$

$e^{1/4} \approx 1.284025$ on calculator

$|1.284025 - 1.25| = 0.034$ error

THE FOLLOWING PROBLEMS ARE NO CALCULATOR

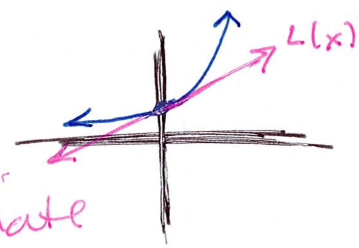
Example 3: Use a linear approximation to estimate the value of $e^{0.1}$. Is that an overestimate or underestimate?

From ex 2

$L(x) = x + 1$

$L(.1) = .1 + 1 = 1.1$

$e^{0.1} \approx 1.1$ underestimate



Example 4: Use a linear approximation to estimate the value of cos 3.2. Is that an overestimate or underestimate?

$f(x) = \cos x$ use $x = \pi$
$y = -1$

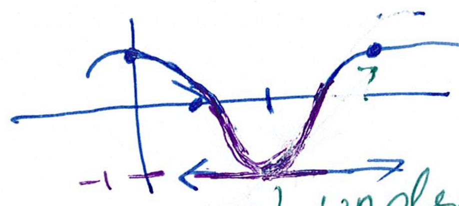
$f'(x) = -\sin x$

$f'(\pi) = -\sin \pi = 0$

$y + 1 = 0(x - \pi)$

$L(x) = -1$

$L(3.2) \approx -1 \approx \cos 3.2$



under estimate

Example 5: Use a linear approximation to estimate the value of $\sqrt{84}$. Is that an overestimate or underestimate?

$f(x) = \sqrt{x}$, $x = 81$, $y = 9$ * Use something close that you know *

$f'(x) = \frac{1}{2\sqrt{x}}$

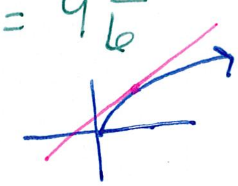
$f'(81) = \frac{1}{2\sqrt{81}} = \frac{1}{18}$

$y - 9 = \frac{1}{18}(x - 81)$

$L(x) = \frac{1}{18}(x - 81) + 9$

$L(84) = \frac{1}{18}(84 - 81) + 9$

$= \frac{3}{18} + 9 = 9\frac{1}{6}$



$\sqrt{84} \approx 9\frac{1}{6}$

overestimate