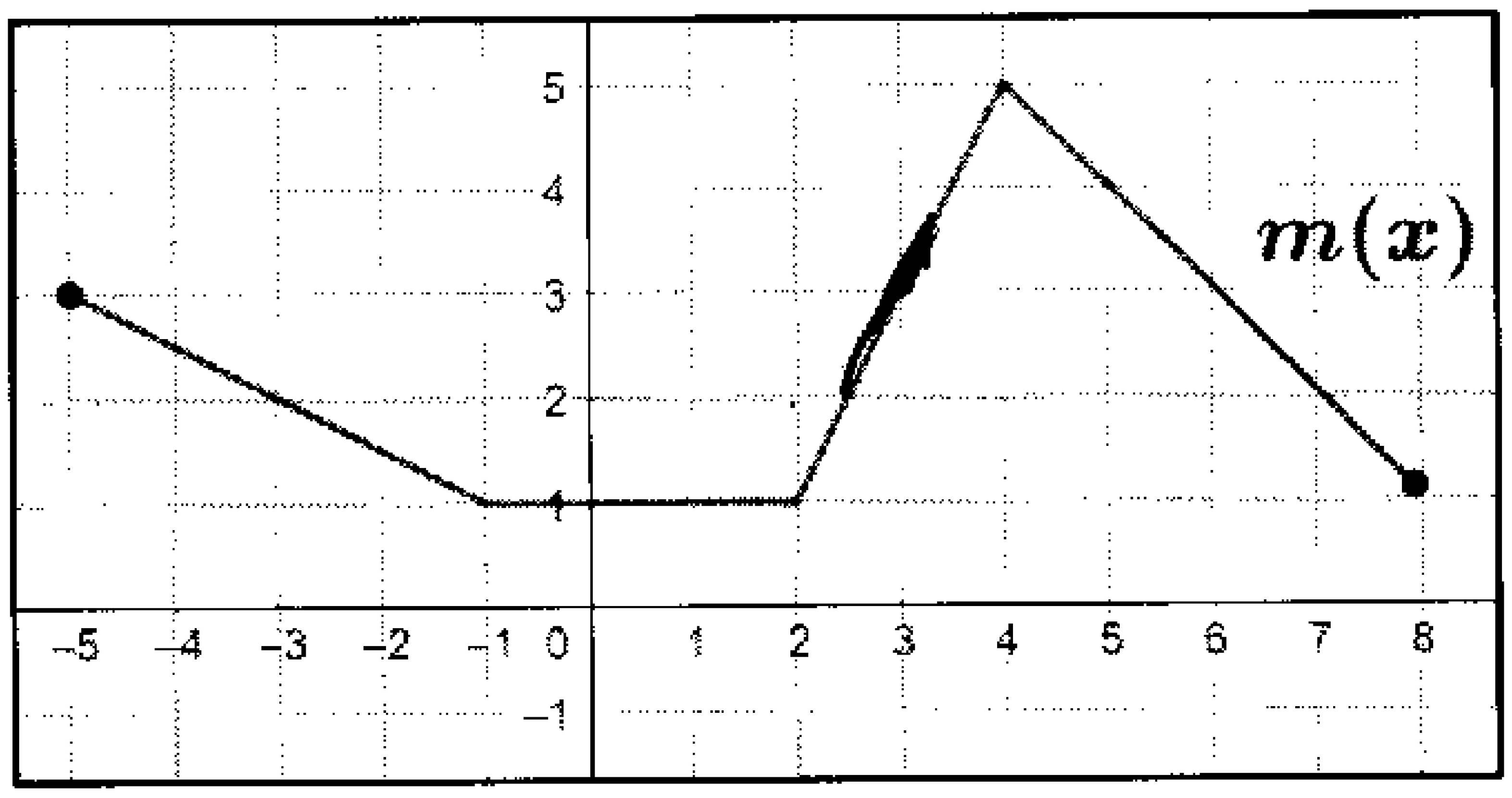
Unit 2 Review Solutions Name:



x	-4	-1	1	3	6
$k(x)$	3	0	-2	4	7
$k'(x)$	-5	2	3	6	-4

$f(x) = x^2 - 2x + 4$ $\qquad g(x) = 2\sin(x) + 4\cos(x)$ $\qquad h(x) = e^x - \frac{2}{x} + ex - 2e$

Directions: The functions $f, g,$ and h are defined above along with the graph of m and a table of selected values for the differentiable functions k and k'. Use this information to find the following.

1. Find the average rate of change of $k(x)$ on the interval $[-4, 6]$.

$(-4, k(-4))$ & $(6, k(6))$ $\qquad \frac{7-3}{6+4} = \frac{4}{10} = \boxed{\frac{2}{5}}$

2. Approximate $k'(2)$.

$\frac{k(3)-k(1)}{3-1} = \frac{4+2}{2} = \frac{6}{2} = \boxed{3}$

3. Find the instantaneous rate of change of k at $x = 3$.

$k'(3) = \boxed{6}$

4. Write an equation of the line tangent to k at $x = 1$.

$(1, -2)$ $\quad k'(1) = 3$ $\qquad \boxed{y+2 = 3(x-1)}$

5. Find the instantaneous rate of change of g at $x = \frac{\pi}{6}$.

$g'(x) = 2\cos x - 4\sin x \Rightarrow g'(\pi/6) = 2\cos\frac{\pi}{6} - 4\sin\frac{\pi}{6} = 2\left(\frac{\sqrt{3}}{2}\right) - 4\left(\frac{1}{2}\right)$
$= \boxed{\sqrt{3} - 2}$

6. Find the instantaneous rate of change of m at $x = \pi$.

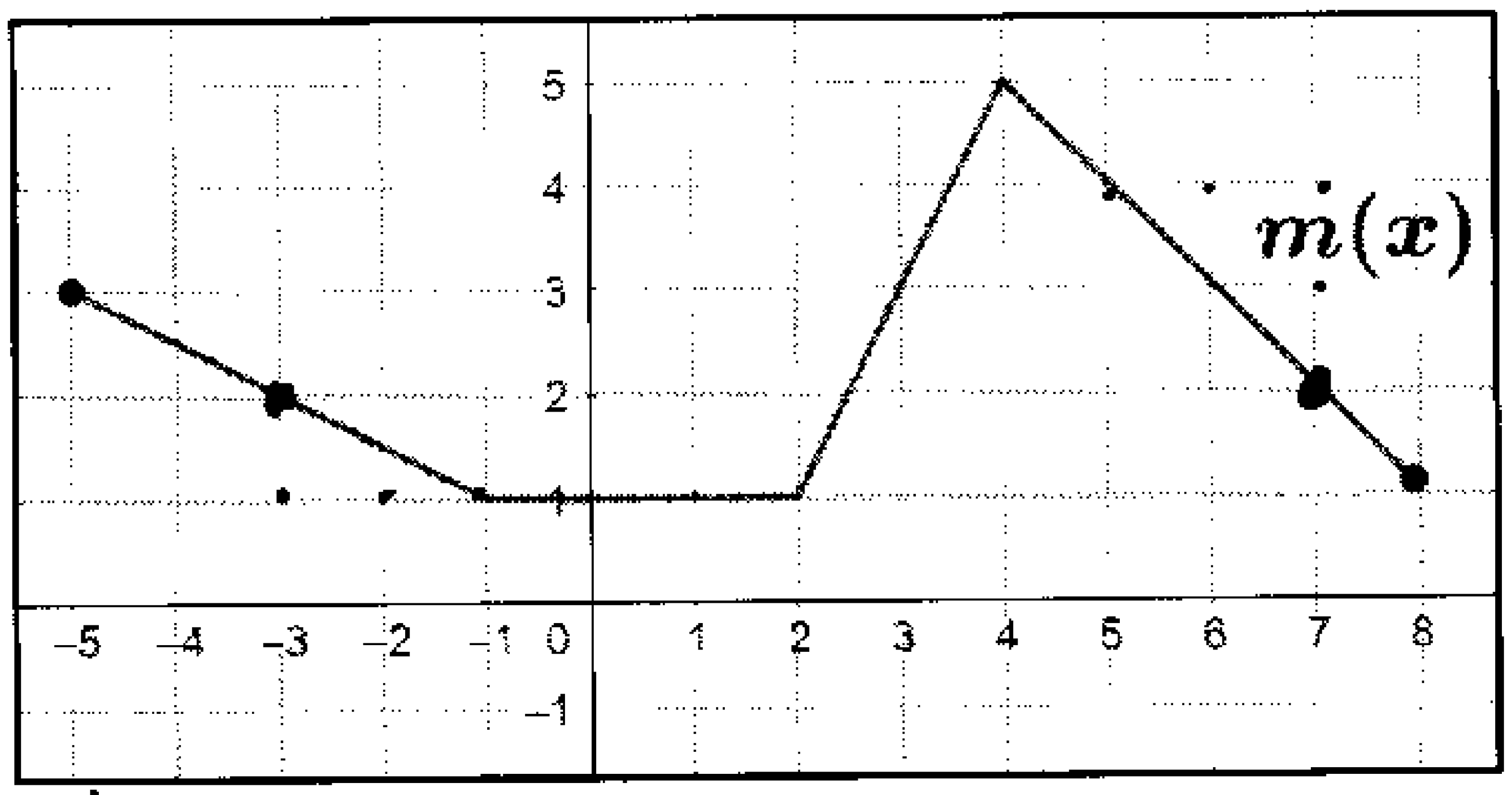
$m'(\pi) = \frac{4}{2} = \boxed{2}$

7. Is there a time c, where $-4 < c < 6$, such that $k(c) = 5$? Give a reason for your answer.

$k(-4) = 3$ and $k(6) = 7$; $3 < 5 < 7$, since K is differentiable k' must be continuous on $(-4, 6)$, so yes IVT guarantees there must be atleast one $k(c) = 5$

8. Is there a time c, where $-4 < c < -1$, such that $k'(c) = -4$? Give a reason for your answer.

$f'(-4) = -5$ and $k'(-1) = 2$; $-5 < -4 < 2$
Since k' is differentiable k' must be continuous on $(-4, -1)$, so IVT guarantees there must be atleast on $k'(c) = 4$



x	−4	−1	1	3	6
$k(x)$	3	0	−2	4	7
$k'(x)$	−5	2	3	6	−4

9−6+4

$f(x) = x^2 - 2x + 4$ $\quad g(x) = 2\sin(x) + 4\cos(x)$ $\quad h(x) = e^x - \frac{2}{x} + ex - 2e$

$f'(x) = 2x - 2$

Directions: The functions $f, g,$ and h are defined above along with the graph of m and a table of selected values for the differentiable functions k and k'. Use this information to find the following.

9. Find the average rate of change of $f(x)$ on the interval $[-1, 2]$.

$$\frac{f(2)-f(-1)}{2+1} = \frac{(2^2-4+4)-((-1)^2+2+4))}{3} = \frac{4-7}{3} = \boxed{-1}$$

10. Find the average rate of change of $m(x)$ on the interval $[-3, 7]$.

$$\frac{m(7)-m(-3)}{7+3} = \frac{2-2}{10} = \boxed{0}$$

11. Let $p(x) = 2m(x) - x^3$. Find $p'(-3)$.

$p'(x) = 2m'(x) - 3x^2$; $p'(-3) = 2m'(-3) - 3(-3)^2$
$= 2(-\frac{1}{2}) - 27 = \boxed{-28}$

12. Find $h'(x)$.

$$\boxed{h'(x) = e^x + \frac{2}{x^2} + e + \cancel{0}}$$

13. Let $q(x) = m(x)k(x)$. Find $q'(1)$.

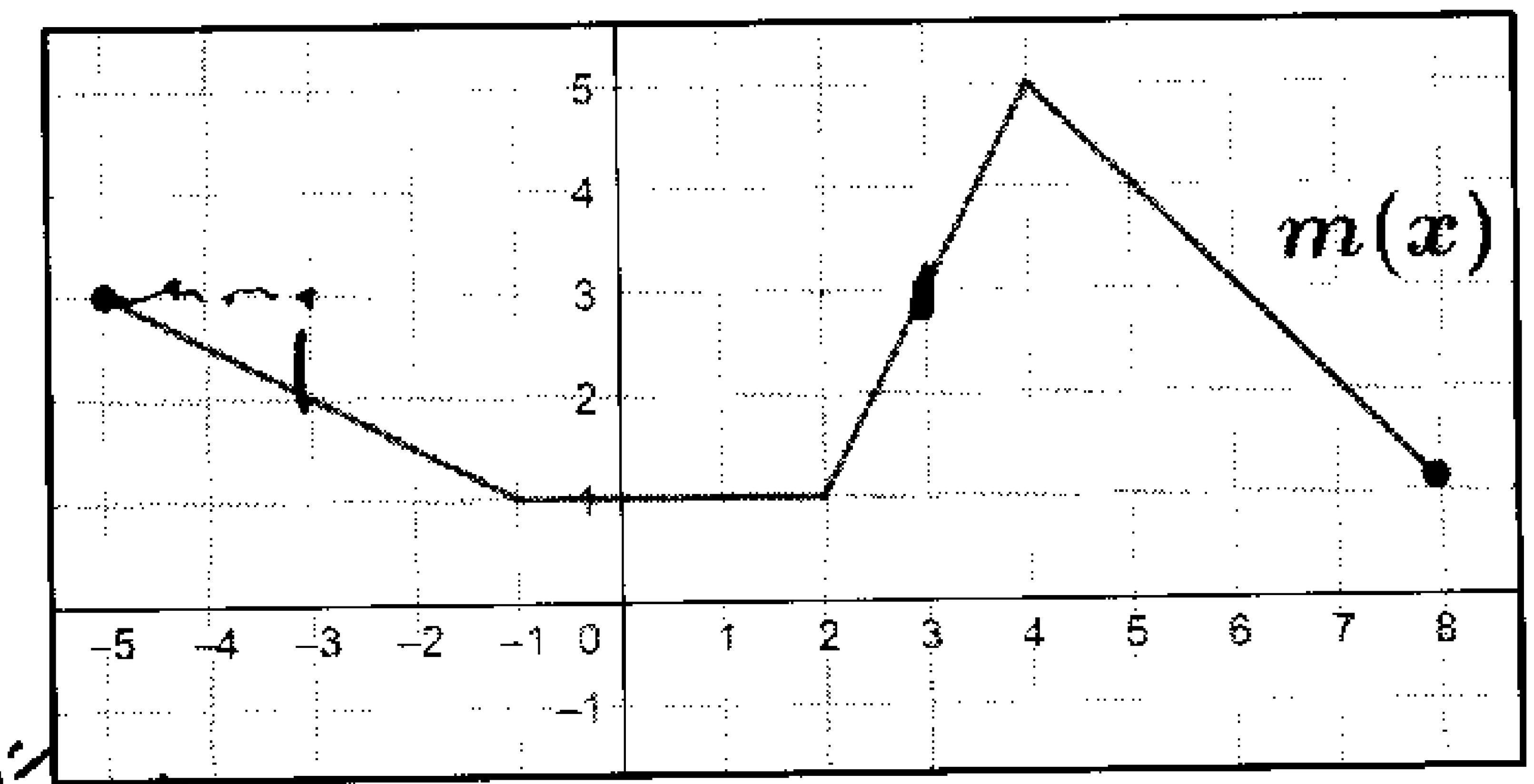
$q'(x) = m(x) \cdot k'(x) + k(x) \cdot m'(x)$
$q'(1) = m(1) \cdot k'(1) + k(1) \cdot m'(1) = 1 \cdot 3 + (-2) \cdot 0 = 3$

14. Let $r(x) = \frac{f(x)}{k(x)}$. Find $r'(3)$.

$$r'(3) = \frac{k(3) \cdot f'(3) - f(3) \cdot k'(3)}{[k(3)]^2} = \frac{4 \cdot 4 - 7 \cdot 6}{16} = \frac{16-42}{16} = \frac{-26}{16} = \boxed{\frac{-13}{8}}$$

15. Find the slope of the line tangent to the graph of m at $x = -4$.

$$m'(-4) = \boxed{\frac{-1}{2}}$$



$f(4)=$
$16-8+4$

x	-4	-1	1	3	6
$k(x)$	3	0	-2	4	7
$k'(x)$	-5	2	3	6	-4

$h'(x) = e^x + \frac{2}{x^2} + e$

$f(x) = x^2 - 2x + 4$ $\qquad g(x) = 2\sin(x) + 4\cos(x)$ $\qquad h(x) = e^x - \frac{2}{x} + ex - 2e$

$f'(x) = 2x - 2$ $\qquad g'(x) = 2\cos x - 4\sin x$

Directions: The functions $f, g,$ and h are defined above along with the graph of m and a table of selected values for the differentiable functions k and k'. Use this information to find the following.

16. For which values of x on the open interval $(-5, 8)$ is $m(x)$ not differentiable?

$\boxed{x = -1, 2, 4}$ $m(x)$ has corners

17. Write an equation for the line tangent to h at $x = 1$.

$h'(1) = e^1 + 2 + e = 2e + 2$

$(1, e^1 - 2 + e - 2e) \Rightarrow (1, -2)$

$\boxed{y + 2 = (2e+2)(x-1)}$

18. Let $w(x) = \frac{m(x)}{2x}$. Find $w'(-3)$.

$w'(x) = \frac{(2x)\cdot m'(x) - m(x)\cdot 2}{4x^2} =$ $\quad w'(-3) = \frac{-6\cdot\frac{-1}{2} - 2\cdot 2}{4(9)} = \boxed{\frac{-1}{36}}$

19. Let $L(x) = 3f(x) + \frac{k(x)}{2}$. Find $L'(3)$.

$L'(x) = 3\cdot f'(x) + \frac{1}{2}k'(x)$

$L'(3) = 3\cdot 4 + \frac{1}{2}(6) = 12 + 3 = \boxed{15}$

20. Let $s(x) = f(x)\sqrt{x}$. Find $s'(4)$.

$s'(x) = f(x)\cdot\frac{1}{2\sqrt{x}} + \sqrt{x}\cdot f'(x)$

$s'(4) = 12\cdot \frac{1}{4} + \sqrt{4}\cdot 6 = 3 + 12 = \boxed{15}$

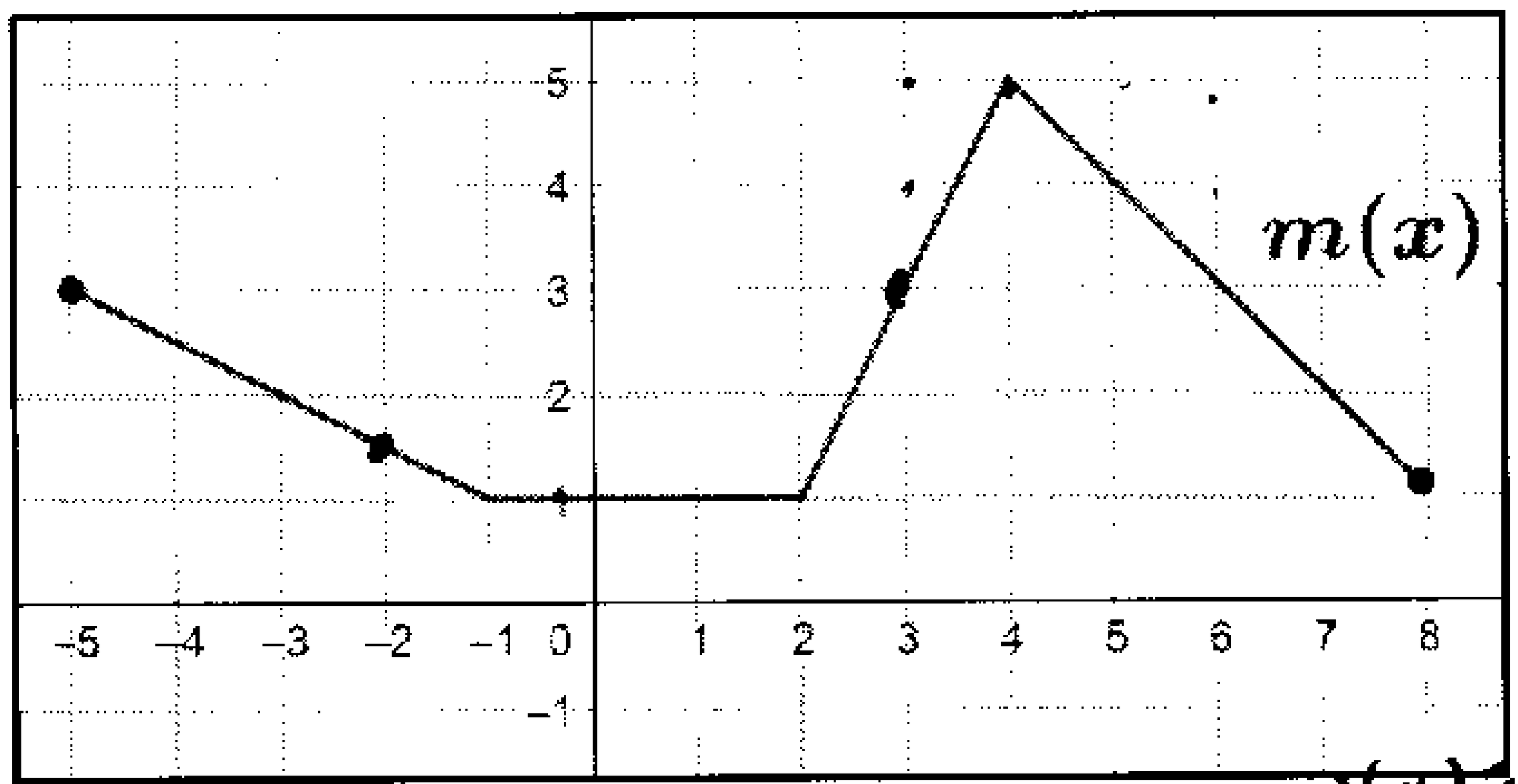
21. Find the slope of the line tangent to the graph of g at $x = \frac{\pi}{4}$.

$g'(\pi/4) = 2\cos\pi/4 - 4\sin(\pi/4)$

$2\cdot\left(\frac{\sqrt{2}}{2}\right) - 4\left(\frac{\sqrt{2}}{2}\right)$

$\sqrt{2} - 2\sqrt{2}$

$\boxed{-\sqrt{2}}$



x	-4	-1	1	3	6
$k(x)$	3	0	-2	4	7
$k'(x)$	-5	2	3	6	-4

$g'(x) = 2\cos x - 4\sin x$; $g''(x) = -2\sin x - 4\cos x$

$f(x) = x^2 - 2x + 4$ $\qquad$ $g(x) = 2\sin(x) + 4\cos(x)$ $\qquad$ $h(x) = e^x - \frac{2}{x} + ex - 2e$

$f'(x) = 2x - 2$ $\qquad$ $0 + 4 = 4$

Directions: The functions $f, g,$ and h are defined above along with the graph of m and a table of selected values for the differentiable functions k and k'. Use this information to find the following.

22. For which value of x in $[-4, 6]$ is the instantaneous rate of change of $k(x)$ equal to the average rate of change of $k(x)$ on the interval $[1, 3]$?

$$K'(x) = \frac{K(3) - K(1)}{3 - 1}$$

$$K'(x) = \frac{4+2}{2} = \frac{6}{2} = 3$$

$$\boxed{x = 1}$$

23. Let $n(x) = \frac{k(x)}{3x+1}$. Find $n'(-1)$.

$$n'(x) = \frac{(3x+1)\cdot K'(x) - K(x)(3)}{(3x+1)^2} = \frac{-2\cdot 2 - 0\cdot 3}{(-3+1)^2} = \frac{-4}{4} = \boxed{-1}$$

24. Let $y = \frac{x^2}{m(x)}$. Find $\left.\frac{dy}{dx}\right|_{x=3}$

$$\frac{dy}{dx} = \frac{m(x)\cdot 2x - x^2\cdot m'(x)}{[m(x)]^2} = \frac{3\cdot 6 - 9\cdot 2}{9} = \boxed{0}$$

25. Let $D(x) = 4\ln(x)\,k(x)$. Find $D'(1)$.

$$D'(x) = 4\ln(x)\cdot K'(x) + K(x)\cdot \frac{4}{x}$$

$$D'(1) = 4\ln 1\cdot 3 + -2\cdot \frac{4}{1} = 0 - 8 = \boxed{-8}$$

Directions: Order the following from least to greatest.

26. $m'(3), m(-2), m'(1)$

$2, 1.5, 0$

$m'(1) < m(-2) < m'(3)$

27. $m'(6), k'(6), f'(6)$

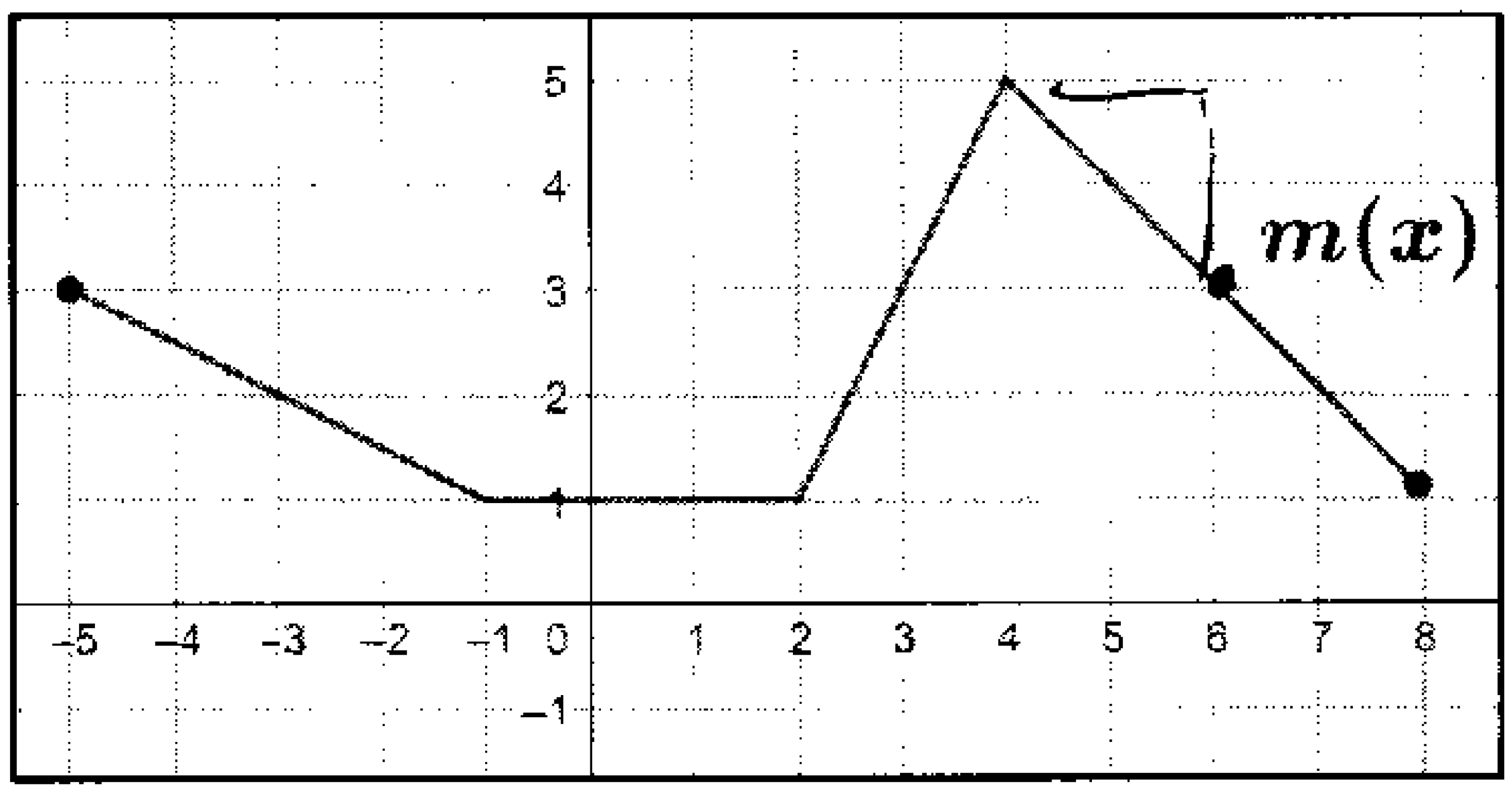
$-1 \quad -4 \quad 10$

$k'(6) < m'(6) < f'(6)$

28. $g(0), g'(0), g''(0)$

$4 \quad 2 \quad -4$

$g''(0) < g'(0) < g(0)$



x	-4	-1	1	3	6
$k(x)$	3	0	-2	4	7
$k'(x)$	-5	2	3	6	-4

$f(x) = x^2 - 2x + 4$ $\qquad$ $g(x) = 2\sin(x) + 4\cos(x)$ $\qquad$ $h(x) = e^x - \dfrac{2}{x} + ex - 2e$

$1-2+4$

Directions: The functions $f, g,$ and h are defined above along with the graph of m and a table of selected values for the differentiable functions k and k'. Use this information to find the following.

29. Let $N(x) = \dfrac{2}{\sqrt{x}} - f(x)$. Find $N'(x)$.

$2 \cdot \frac{1}{\sqrt{x}}$ use reciprocal rule w/ square root rule

$$N'(x) = 2 \cdot \frac{-\left(\frac{1}{2\sqrt{x}}\right)}{x} - f'(x) = \boxed{\frac{-1}{x\sqrt{x}} - f'(x)} \text{ or } \frac{-1}{x^{3/2}} - f'(x)$$

30. Find any value(s) of x where $f(x)$ has a horizontal tangent line.

$f'(x) = 0$

$2x - 2 = 0$

$\boxed{x = 1}$

If it asked for point(s) $(1, 3)$

31. Let $A(x) = 2k(x) + 3x$. For which value(s) of x does the line tangent to the graph of $A(x)$ have a slope of -5?

$A'(x) = 2k'(x) + 3$

$-5 = 2k'(x) + 3$

$k'(x) = -4 \Rightarrow \boxed{x = 6}$

32. Let $B(x) = 6\ln|x| - 5e^x + \dfrac{3}{x}$. Find the slope of the line tangent to the graph of B at $x = -1$.

$$B'(x) = 6 \cdot \frac{1}{|x|} - 5e^x - \frac{3}{x^2} \ ; \ B'(1) = 6 - 5e^{-1} - \frac{3}{(-1)^2}$$

$$= -6 - \frac{5}{e} - 3 = \boxed{-9 - \frac{5}{e}}$$

33. Let $C(x) = 2x^3 - m(x)k(x)$. Find $C'(6)$.

$C'(x) = 6x^2 - [m(x) \cdot k'(x) + k(x) \cdot m'(x)]$

$C'(6) = 6(36) - [3 \cdot -4 + 7 \cdot -1] = 216 - (-19) = \boxed{235}$

34. Let $L(x) = \dfrac{k(x)}{k'(x)}$. If L has a horizontal tangent line at $x = 1$, find $k''(1)$.

$$L'(x) = \frac{k'(x) \cdot k'(x) - k(x) \cdot k''(x)}{[k'(x)]^2}$$

$$0 = [3 \cdot 3 - (-2) \cdot k''(1)] / 3^2 \Rightarrow 0 = \frac{9 + 2k''(1)}{9} \Rightarrow k''(1) = \boxed{\frac{-9}{2}}$$