2021 #5 $\boxed{\text{MEAN } 4.26}$

a) $f(1) + f'(1)(x-1) + \frac{f''(1)}{2!}(x-1)^2$

$\left.\frac{dy}{dx}\right|_{\substack{x=1\\y=4}} = 4 \cdot \ln(1) = 0$

$P_2(x) = 4 + 0(x-1) + \frac{4}{2}(x-1)^2$ +1

$P(2) \approx 4 + 2(2-1)^2 \approx \underline{6}$ +1

b)

x	y	$\Delta y = (y \cdot x \ln x) \cdot (.5)$	New y-value
1	4	$4 \cdot 1 \cdot \ln 1\,(.5) = 0$	$4 + 0 = 4$
1.5	4	$(4 \cdot 1.5 \ln 1.5)(.5) = 3 \ln 1.5$	$4 + 3\ln 1.5$
2			

+1 +1

c) $\frac{dy}{dx} = y(x \ln x)$

$\frac{1}{y}dy = x \cdot \ln x \, dx$ +1

+1 $\underline{\ln|y|} = \int x \ln x \, dx$

Integration by parts

$u = \ln x \quad dv = x\,dx$

$du = \frac{1}{x}dx \quad v = \frac{x^2}{2}$

$= \ln x\left(\frac{x^2}{2}\right) - \int \frac{x^2}{2} \cdot \frac{1}{x} dx$

$= \underline{\frac{x^2}{2} \cdot \ln x - \frac{x^2}{4} + C}$ +1

2021 #5 cont.,

$$\ln|4| = \frac{(1)^2}{2} \cdot \ln(1) - \frac{(1)^2}{4} + C$$

$$\ln 4 = 0 - \frac{1}{4} + C$$

$$C = \ln(4) + \frac{1}{4} \quad +1$$

$$y = e^{\frac{x^2}{2}\ln x - \frac{x^2}{4} + \ln(4) + \frac{1}{4}}, \quad x > 0 \quad +1$$

2019 BC #4 $\boxed{\text{MEAN } 3.56}$

a) $V = \pi r^2 h$; $r = 1$ ft is constant

$V = \pi(1)^2 h = \pi h$

$\frac{dV}{dt} = \pi \cdot \frac{dh}{dt}$ +1 ; Given $\frac{dh}{dt} = \frac{-1}{10}\sqrt{h}$

$\left.\frac{dV}{dt}\right|_{h=4} = \pi \cdot \frac{-1}{10}\sqrt{4} = \underline{\frac{-\pi}{5} \text{ ft}^3/\text{sec}}$ +1

b) $\frac{d^2h}{dt^2} = \frac{-1}{10} \cdot \underline{\frac{1}{2\sqrt{h}}} \cdot \frac{dh}{dt} = \underline{\frac{-1}{20\sqrt{h}} \cdot \frac{-1}{10}\sqrt{h} = \frac{1}{200}}$

+1 +1

When the height of the water is 3ft, the rate of change of the height of the water w/ respect to time in <u>increasing</u> +1 <u>because $\frac{d^2h}{dt^2} > 0$.</u>

c) $\int \frac{1}{\sqrt{h}}\, dh = \int \frac{-1}{10}\, dt$ +1

$2\sqrt{h} = \frac{-1}{10}t + C$ +1

$2\sqrt{5} = \frac{-1}{10}(0) + C \Rightarrow C = 2\sqrt{5}$ +1

$2\sqrt{h} = \frac{-1}{10}t + 2\sqrt{5}$

$h = \left(\frac{-1}{20}t + \sqrt{5}\right)^2$ +1

2017 #4 No Calculator $\boxed{\text{MEAN } 3.13}$

a) $t=0,\ H(0)=91$

$$\frac{dH}{dt} = -\frac{1}{4}(H-27) \Rightarrow H'(0) = -\frac{1}{4}(91-27)$$
$$= -16 \quad +1$$

$$y - 91 = -16(t-0)$$

+1 $\boxed{\text{or}}$ $H(t) = -16t + 91$

$$H(3) = -16(3)+91 = -48+91 = \underline{43^\circ C} \quad +1$$

b) $\frac{d^2H}{dt^2}$ or $H''(t) = -\frac{1}{4}(H'-0) = -\frac{1}{4}H' = -\frac{1}{4}\cdot -\frac{1}{4}(H-27)$

+1 answer w/ reason

$$H''(t) = \frac{1}{16}(H-27)$$

$H''(3) = \frac{1}{16}(H-27)$, $t=3$, $H>27$ Given for all $t>0$.

$H''(3) > 0$, therefore $H(t)$ is concave up at $t=3$, thus part (a) is an <u>underestimate</u>.

c) $\frac{dG}{dt} = -(G-27)^{2/3}$

$$\frac{1}{(G-27)^{2/3}}dG = -dt \quad +1$$

$$\int \frac{1}{(G-27)^{2/3}}dG = \int -1\cdot dt$$

$$+1 \quad \underline{3\cdot(G-27)}^{-2/3+3/3} = \underline{-t+C}\ ;\ (0,91)$$

$$3(91-27)^{1/3} = 0 + C \Rightarrow \underline{C=12} \ +1$$

$$3(G-27)^{1/3} = -t+12$$

$$(G-27)^{1/3} = \frac{12-t}{3}$$

$$G-27 = \left(\frac{12-t}{3}\right)^3$$

$$+1 \quad G(t) = \underline{27 + \left(\frac{12-t}{3}\right)^3} \text{ for } 0 \le t < 10$$

should include

$$G(3) = 27 + (3)^3$$
$$= 54^\circ C \quad +1$$

DIFFEQ PACKET

$\boxed{\text{MEAN } 4.68}$

2016 #4

a) $\dfrac{dy}{dx} = x^2 - \dfrac{1}{2}y$

$\dfrac{d^2y}{dx^2} = 2x - \dfrac{1}{2} \cdot \dfrac{dy}{dx}$ +1

$\dfrac{d^2y}{dx^2} = 2x - \dfrac{1}{2}\left(x^2 - \dfrac{1}{2}y\right)$ +1

b) $\left.\dfrac{dy}{dx}\right|_{(-2,8)} = (-2)^2 - \dfrac{1}{2}(8)$

$= 4 - 4 = 0$

You cannot use a sign chart because because we don't know an x + y on each side of -2. So, we have to look at 2nd derivative (2nd derivative test)

$\left.\dfrac{d^2y}{dx^2}\right|_{(-2,8)} = 2(-2) - \dfrac{1}{2}\left((-2)^2 - \dfrac{1}{2}(8)\right)$

$-4 - 0 = -4$

f has a relative max +1 because $\dfrac{dy}{dx} = 0$ +1 and $\dfrac{d^2y}{dx^2} < 0$ at $(-2,8)$ which means f is concave down

2016 #4

c) $\lim\limits_{x\to -1}\left(g(x)-2\right)=0$ and $\lim\limits_{x\to 1} 3(x+1)^2=0$

must show separately in order to use

+1 L'Hopital's Rule

*$g'(-1)$; $(-1,2)$
$=(-1)^2-\frac{1}{2}(2)=0$

$$\lim_{x\to -1}\left(\frac{g(x)-2}{3(x+1)^2}\right)=\lim_{x\to -1}\left(\frac{g'(x)}{6(x+1)}\right)$$

$\lim\limits_{x\to -1} g'(x)=\underline{0}$ and $\lim\limits_{x\to -1} 6(x+1)=0$

$$\lim_{x\to -1}\frac{g'(x)}{6(x+1)}=\lim_{x\to -1}\frac{g''(x)}{6}=-\frac{\underline{2}}{6}=-\frac{1}{3}$$

+1

*$g''(-1)$; $(-1,2)$
$=2(-1)-\frac{1}{2}\left[(-1)^2-\frac{1}{2}(2)\right]$ from part(a)
$=-2$

d) $\Delta x=\frac{1-0}{2}=\frac{1}{2}$

x	y	$\Delta y=\left(x^2-\frac{1}{2}y\right)\cdot\left(\frac{1}{2}\right)$	New y
0	2	$=\left[0^2-\frac{1}{2}(2)\right]\cdot\left(\frac{1}{2}\right)=\frac{-1}{2}$	$2+-\frac{1}{2}=\underline{\frac{3}{2}}$
$\frac{1}{2}$	$\underline{\frac{3}{2}}$	$\left[\left(\frac{1}{2}\right)^2-\frac{1}{2}\left(\frac{3}{2}\right)\right]\cdot\left(\frac{1}{2}\right)$ $=\left(\frac{1}{4}-\frac{3}{4}\right)\left(\frac{1}{2}\right)=\frac{-1}{4}$	$\frac{3}{2}+\frac{-1}{4}=\frac{6}{4}-\frac{1}{4}=\underline{\frac{5}{4}}$
1	$\underline{\frac{5}{4}}$		