2019 # 2

a) $A = \frac{1}{2}\int_0^{\sqrt{\pi}} (r(\theta))^2 \, d\theta = 3.5343$

+1 +1

b) $\frac{1}{\sqrt{\pi} - 0}\int_0^{\sqrt{\pi}} r(\theta) \, d\theta = 1.5799$

+1 +1

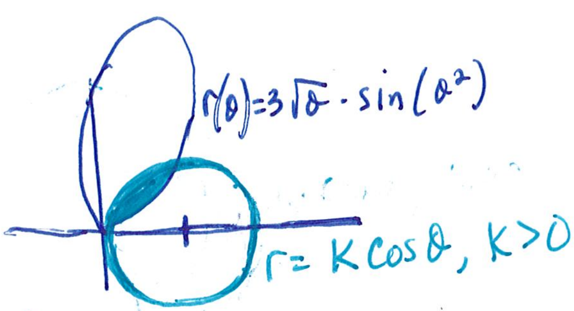
c) *Converting rectangular to Polar $\theta = \tan^{-1}\left(\frac{y}{x}\right)$ when $x > 0$

See 9.7 Polar Review Notes

$\frac{1}{2}\int_0^{\tan^{-1}(m)} (r(\theta))^2 \, d\theta = \frac{1}{2} \cdot \frac{1}{2}\int_0^{\sqrt{\pi}} (r(\theta))^2 \, d\theta$

+1 +1 +1

d)



$\lim_{K \to \infty} A(K) = \frac{1}{2}\int_0^{\pi/2} (r(\theta))^2 \, d\theta$

$= 3.3244$

As $K \to \infty$, the circle becames larger and will encompass all points to right $\pi/2$

2018 #5 No CALC

a) $A = \frac{1}{2}\int_{\pi/3}^{5\pi/3}\left(4^2 - (3+2\cos\theta)^2\right)d\theta$

or $A = \int_{\pi/3}^{\pi}\left(4^2 - (3+2\cos\theta)^2\right)d\theta$
+1 +1
+1

b) $y = (3+2\cos\theta)\cdot\sin\theta$ $\qquad x = (3+2\cos\theta)\cdot\cos\theta$

$\frac{dy}{d\theta} = (3+2\cos\theta)\cdot\cos\theta + \sin\theta(-2\sin\theta)$ $\qquad \frac{dx}{d\theta} = (3+2\cos\theta)\cdot -\sin\theta + \cos\theta(-2\sin\theta)$

+1 $\frac{dy}{d\theta} = 3\cos\theta + 2\cos^2\theta - 2\sin^2\theta$ $\qquad \frac{dx}{d\theta} = -3\sin\theta - 4\sin\theta\cos\theta$

+1 $\left.\frac{dy}{dx}\right|_{\theta=\pi/2} = \boxed{\frac{3\cos\pi/2 + 2\cos^2\pi/2 - 2\sin^2\pi/2}{-3\sin\pi/2 - 4\sin\pi/2\cos\pi/2}}$ STOP +1

$= \frac{2}{3}$

c) $\frac{dr}{dt} = -2\sin\theta\cdot\frac{d\theta}{dt}$ +1

$\frac{d\theta}{dt} = \frac{dr}{dt}\cdot\frac{1}{-2\sin\theta}$ +1

$\left.\frac{d\theta}{dt}\right|_{\theta=\pi/3} = \boxed{3\cdot\frac{1}{-2\sin\frac{\pi}{3}}}$ STOP but include units
+1

$= 3\cdot\frac{1}{-\sqrt{3}} = -\sqrt{3}$ radians/sec

2017 #2

a) $A = \underline{\frac{1}{2}\int_0^{\pi/2} (f(\theta))^2\, d\theta} = 0.6484$ +1

+1

b) $\int_0^{K} \left[(g(\theta))^2 - (f(\theta))^2\right] d\theta = \int_K^{\pi/2} \left[(g(\theta))^2 - (f(\theta))^2\right] d\theta$

+1 +1

c) $w(\theta) = g(\theta) - f(\theta)$ +1

outer - inner

Avg Value of $w(\theta) = \dfrac{1}{\frac{\pi}{2} - 0} \displaystyle\int_0^{\pi/2} w(\theta)\, d\theta$ +1

$= 0.4854$ +1

d) $g(\theta) - f(\theta) = 0.4854$

+1 $\theta = 0.5177$; $[0, \pi/2]$

$w'(0.5177) < 0 \therefore w(\theta)$ is decreasing

+1 at $\theta = 0.5177$

2014 #2

a) $A = \frac{1}{2}\int_0^{\pi/2} (3-2\sin(2\theta))^2 d\theta + \frac{1}{2}\int_{\pi/2}^{\pi} (3)^2 d\theta$

$+1$ $\quad$ $+1$

$= 9.7079$ $+1$

b) $x = (3-2\sin(2\theta))\cos\theta$ $+1$

$\left.\frac{dx}{d\theta}\right|_{\theta=\pi/6} = -2.3660$ $+1$

c) $D = 3-(3-2\sin(2\theta)) = 2\sin(2\theta)$ $+1$

$\left.\frac{dD}{d\theta}\right|_{\theta=\pi/3} = -2$ $+1$

d) $r = 3-2\sin(2\theta)$

$\frac{dr}{dt} = -4\cos(2\theta)\cdot\frac{d\theta}{dt}$ $+1$

$\left.\frac{dr}{dt}\right|_{\theta=\pi/6} = -4\cos(2\cdot\pi/6)\cdot 3$

$= -6$ $+1$